Disorder fosters chimera in an array of motile particles

L. A. Smirnov ,[1,2] M. I. Bolotov ,[1] G. V. Osipov ,[1] and A. Pikovsky [3,1]
[1]*Department of Control Theory, Research and Education Mathematical Center "Mathematics for Future Technologies", Nizhny Novgorod State University, Gagarin Avenue 23, 603022, Nizhny Novgorod, Russia*
[2]*Institute of Applied Physics of the Russian Academy of Sciences, Ul'yanov Street 46, 603950, Nizhny Novgorod, Russia*
[3]*Institute of Physics and Astronomy, Potsdam University, 14476 Potsdam-Golm, Germany*

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We consider an array of nonlocally coupled oscillators on a ring, which for equally spaced units possesses a Kuramoto–Battogtokh chimera regime and a synchronous state. We demonstrate that disorder in oscillators positions leads to a transition from the synchronous to the chimera state. For a static (quenched) disorder we find that the probability of synchrony survival depends on the number of particles, from nearly zero at small populations to one in the thermodynamic limit. Furthermore, we demonstrate how the synchrony gets destroyed for randomly (ballistically or diffusively) moving oscillators. We show that, depending on the number of oscillators, there are different scalings of the transition time with this number and the velocity of the units.

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I. INTRODUCTION

Chimera patterns, discovered by Kuramoto and Battogtokh (KB) almost 20 years ago [1], continue to be in the focus of theoretical and experimental studies (see recent reviews [2,3]). Chimera is a spatial pattern in an oscillatory medium, where some subset of oscillators is synchronous and forms an ordered patch, while other oscillators in a disordered patch are asynchronous. There is bistability in the classical KB setup of nonlocally coupled oscillators on a ring: a chimera pattern coexists with a fully synchronized, homogeneous in space state. In this bistable situation, one should specially prepare initial conditions to observe chimera, because the basin of the synchronous state is relatively large. Moreover, in a finite population (i.e., for a finite number of oscillators on the ring), the synchronous state appears to be a global attractor: the chimera state is a transient, slightly irregular state, which has a lifetime exponentially growing with the number of oscillators [4]. This paper demonstrates that disorder in the KB setup fosters the opposite: synchronous state disappears, while chimera remains stable.

The effect of disorder on chimera has been explored in several recent publications. S. Sinha [5] studied different models of coupled maps and coupled oscillators, and demonstrated that with the introduction of time-varying random links to the network of interactions, a chimera is typically destroyed, and the synchronous state establishes. In Ref. [6], the effect of random links addition on the chimera state in coupled FitzHugh–Nagumo oscillators has been studied. It has been demonstrated that although for a weak disorder, chimera survives, it becomes destroyed if the disorder is strong. We mention here also Ref. [7], in which disorder has been explored for a variant of a chimera state not in a spatially extended state, but for two globally coupled subpopulations of oscillators [8].

Another way to include disorder in the setup of coupled oscillators is to assume that the units are motile particles, possibly with randomness in their motion. There are two ways in constructing such models: (i) one can assume that the oscillatory dynamics of the elements does not influence their motion, so that there is only the influence of the positions of the units on their oscillatory dynamics (see Ref. [9]), and (ii) there is a mutual interaction between motion and internal dynamics (see, e.g., Refs. [10,11]). For example, for locally coupled phase oscillators randomly moving on one-dimensional lattice [9], motility has been shown to promote a synchronous state. For two-dimensional motions, the authors of Ref. [12] observed that there is a resonance range of random velocities, for which the transition to synchrony is extremely slow. The authors of Ref. [13] explored one-dimensional lattice with local delayed coupling, the motion of particles was modeled by random exchanges of positions of nearest neighbors; in this setup, a persistent chimera was observed in some range of parameters. Close in terms of the formulation of the problem is a recent study by Wang *et al.* [14]. In this work, 128 diffusive particles on a line have been considered. Each particle is a phase oscillator, and the coupling is nonlocal with a cos-shaped kernel (like in the chimera studies [15]). Depending on the parameter of diffusion and coupling, both transitions from a chimera to a synchronous state and from a synchronous state to a chimera have been observed. Finally, we mention an important experimental setup where moving particles synchronize. Prindle *et al.* [16] considered a population of 2.5 millions of *Escherichia coli* bacterial cells equipped with genetically engineered clocks, and observed their synchronization under conditions where these cells were transported in a microfluidic device, with a coupling through a chemical messenger.

In this paper, we explore the effect of disorder in particles' positions on the properties of the "classical" KB chimera [1]. We consider quenched disorder (random fixed position of the particles on the ring), and dynamical disorder (diffusive or ballistic motion of the particles). Below we restrict our attention to the case of slow motions, which can be explored by comparing with the quenched case. We will show, that the number of particles is the essential parameter governing the dynamics, and establish scaling properties in dependence on the parameters determining the particles velocities, and on their number.

The paper is organized as follows. First, we introduce the model in Sec. II. The case of quenched disorder is considered in Sec. III. Properties of motile oscillators are considered in Sec. IV. Finally, we conclude and discuss the results in Sec. V.

II. BASIC MODELS

We introduce our basic model as a generalization of the Kuramoto–Battogtokh setup [1] for a ring of coupled phase oscillators (particles). In contradistinction to Ref. [1], where equally spaced positions of the oscillators were assumed, we consider general positions $0 \leqslant x_i < 1$ for N oscillators on the ring. The coupling is distance-dependent,

$$\dot{\varphi}_i = \frac{1}{N}\sum_{j=1}^{N} G(x_j - x_i)\sin(\varphi_j - \varphi_i - \alpha), \tag{1}$$

according to the kernel

$$G(x) = \frac{\kappa \cosh[\kappa(|x| - 0.5)]}{2\sinh\frac{\kappa}{2}}, \tag{2}$$

which is a generalization of the exponential kernel adopted in Ref. [1] to account for periodic boundary conditions on the ring. Parameter κ determines the effective range of coupling, parameter α is the phase shift in coupling.

For positions of the particles x_i, we explore three models in this paper.

(1) **Quenched disorder:** Here the positions x_i of the particles are fixed, taken as independent random variables with a uniform distribution on a ring.

(2) **Diffusion of the particles:** Here the particles are driven by independent white Gaussian noise terms, leading to their diffusion (with diffusion constant σ^2)

$$\dot{x}_i = \sigma\xi_i(t), \quad \langle\xi_i(t)\rangle = 0, \quad \langle\xi_i(t)\xi_j(t')\rangle = \delta_{ij}\delta(t - t'). \tag{3}$$

(3) **Ballistic motion of the particles:** Here the particles move with constant fixed random velocities v_i. Below we consider velocities as i.i.d. Gaussian random variables with standard deviation μ.

In this paper we restrict ourselves to the cases of slow motion of the particles, i.e., to the cases of small parameters σ and μ.

III. QUENCHED DISORDER

A. Observation of a transition to chimera

We start with the case of quenched disorder. Here the only parameter is the number of the particles N. In the thermodynamic limit $N \to \infty$, one does not expect any deviation of the dynamics of disordered sets from the dynamics of ordered configurations, because in both cases in the limit $N \to \infty$ one obtains a system of integrodifferential equations for the distribution of phases $\varphi(x, t)$:

$$\partial_t\varphi(x, t) = \int_0^1 d\tilde{x}\; G(\tilde{x} - x)\sin[\varphi(\tilde{x}, t) - \varphi(x, t) - \alpha]. \tag{4}$$

The population of phase oscillators Eq. (4), as has been first demonstrated by Kuramoto and Battogtokh [1], possesses two attracting states: (i) a fully synchronous state $\varphi(x, t) = \psi(t)$ and (ii) a spatially inhomogeneous chimera state with domains of synchrony (neighboring phases are closed to each other) and of asynchrony (neighboring phases are taken from a certain probability distribution). Finite-size effects for a *regular* distribution of oscillators on the ring have been explored by Wolfrum and Omelchenko [4]. The synchronous state is still stable for any N, but the chimera state appeared to be a chaotic supertransient, which lives for an exponentially growing with N time interval, but eventually goes into the synchronous state.

Our main observation is that the opposite happens for an *irregular* distribution of oscillators on the ring. Namely, an initial synchronous state may become destroyed for finite N, while the chimera state is stable. We illustrate a transition from a synchronous to a chimera regime in Fig. 1.

Qualitatively, destruction of the synchronous state due to disorder is similar to desynchronization in disordered oscillator lattices first described by Ermentrout and Kopell [17]. At large enough disorder a synchronous state in the lattice disappears due to a saddle-node bifurcation. In our setup we cannot directly apply the theory in Ref. [17], because we have a ring with a long-range coupling. Furthermore, the theory in Ref. [17] is restricted to the case $\alpha = 0$, while in our setup parameter α is close to $\pi/2$.

B. Statistical evaluation

In Fig. 2 we present a direct statistical evaluation of the probability for synchrony to occur. The numerical experiment has been performed as follows: for a configuration of random positions of oscillators x_i, Eqs. (1) were solved starting from the state with all phases being equal $\varphi_1 = \varphi_2 = \ldots = \varphi_N$. If particles evolve toward a steady rotating state, where all the instantaneous frequencies are equal, then the configuration is considered as a synchronous one. Otherwise, if in the set of oscillators phase slips appear, then the configuration is considered as a nonsynchronous (chimera). The numerical procedure was as follows. The calculations started from the initial state $\varphi_1 = \varphi_2 = \ldots = \varphi_N$. During the evolution the difference of the instantaneous frequencies $\Delta\Omega$ between the most fast and the most slow oscillator in the array was calculated. If this quantity exceeded the threshold $\Delta\Omega_c = 0.2$, then this configuration was considered as a nonsynchronous one. If during the time interval $\Delta T = 200$ the difference remained below the threshold, then the configuration was considered as a synchronous one (this time interval is approximately two times longer than the average time for the transition to chimera at $N = 16\,384$). Typical values of the frequency difference for a synchronous configuration at $\Delta T = 200$ were $\Delta\Omega \lesssim 10^{-4}$. Many runs with different random positions have been sampled to achieve statistical results presented in the Fig. 2. One can see that while the probability to observe synchrony is very low for relatively small N (in fact, for $N = 256$ no synchronous case out of 10^4 runs has been observed), it becomes high for $N \gtrsim 8\,192$. This confirms the qualitative picture of the local stability of the

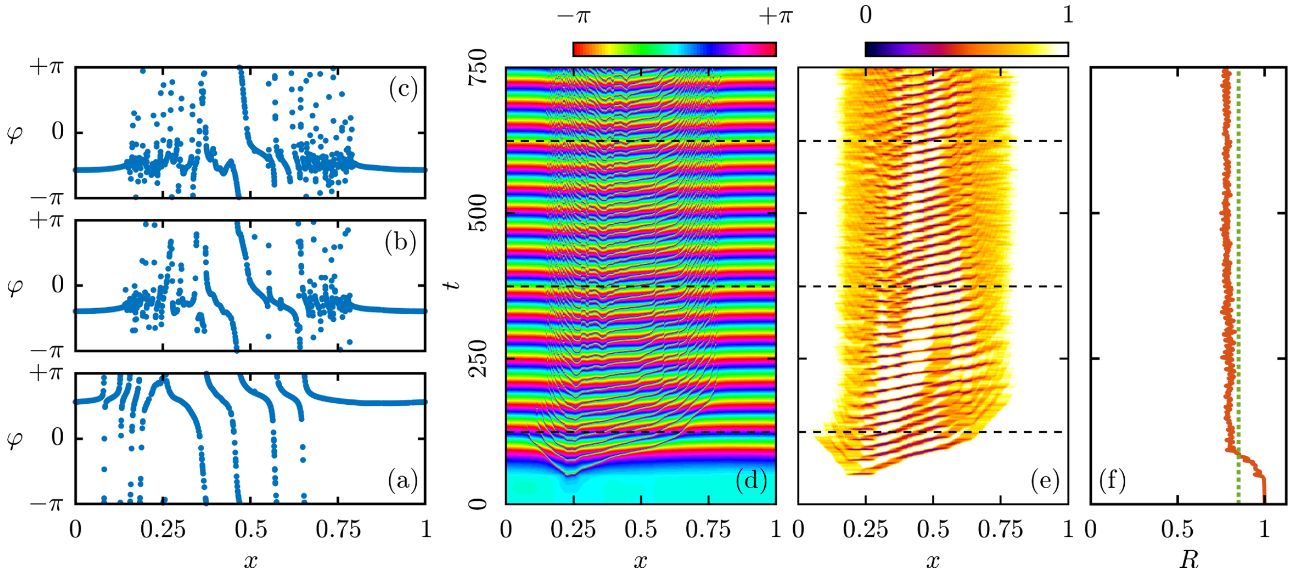

FIG. 1. Illustration of the transition synchrony → chimera for quenched disorder and $N = 1024$ (other parameters: $\kappa = 4$, $\alpha = 1.457$). The particles are placed randomly on the circle, and their phases are initially equal. Panels (a, b, and c): snapshots of phase distributions $\varphi(x, t)$ at (a) $t = 125$, (b) $t = 375$, and (c) $t = 625$. One can see how the synchronous state is destroyed in the presence of spatial disorder. First, phase slips in a certain region of space occur. Further, clusters with the highest phase gradient begin to break down, which leads to the formation of intervals with an irregular spatial distribution of the dynamic variable $\varphi(x, t)$. After that the system goes to a chimera state. Panel (d): spatiotemporal dynamics of the phases $\varphi(x, t)$. Panel (e): absolute value of the local (calculated for 17 neighbors) order parameter $Z(x_i, t) = (17)^{-1} \sum_{j=-8}^{8} \exp[i\varphi_{i+j}]$, additionally averaged over the time interval of 3 time units. White regions correspond to synchrony. Black dashed lines denote the moments in time for which snapshots of the phases $\varphi(x, t)$ are presented on panels (a, b, and c). Panel (f): the dynamics of the global order parameter $R(t) = |N^{-1} \sum_{j=1}^{N} \exp[i\varphi_j]|$. It is clearly seen how the transition from the initially synchronous regime with $R = 1$ to the chimera state with $R \approx 0.79$ occurs. The green dashed line shows the value $R = 0.85$, which is further taken as a criterion that determines the time of destruction of the synchronous mode.

synchronous state at $N \to \infty$. We stress here that we do not consider very small systems with a few oscillators.

C. Analytic estimate of probability of the existence of stable synchrony

Here we give a semianalytic estimate for the probability to observe a synchronous state in a disordered array. Instead of performing a rigorous bifurcation analysis, we first estimate

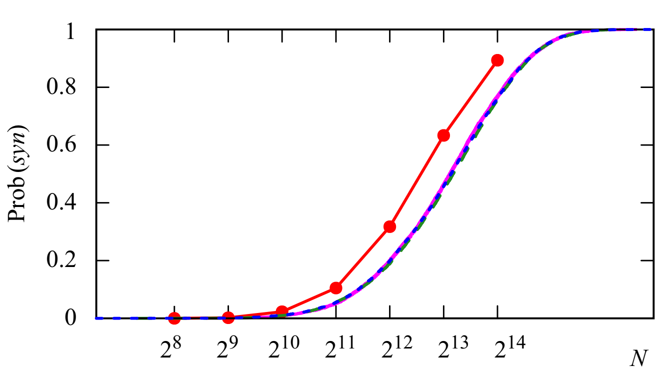

FIG. 2. Red dots: probabilities of existence of a synchronous state from direct numerical simulations. Curves: rescaled cumulative distributions of the minimum of field H, for $N = 128$ (green curve), $N = 256$ (blue), and $N = 512$ (magenta). These curves are drawn with help of Eq. (7) and practically overlap, which confirms the validity of the scaling $\sim N^{1/2}$.

(approximately) a critical fluctuation of the local coupling strength at which the synchronous state disappears. The coupling strength for oscillator x_k is defined, according to Eq. (1), as

$$H(x_k) = \frac{1}{N} \sum_{j=1}^{N} G(x_j - x_k).$$

Let us first consider a fully synchronous state in the ordered lattice (i.e., with regular positions of the oscillators). In this case $H(x_k) = 1$, due to the adopted normalization of the kernel. Setting $\varphi_1 = \varphi_2 = \ldots = \varphi_N = \Phi$, one obtains for the phases a uniform synchronous rotation

$$\dot{\Phi} = -\sin\alpha.$$

Let us now take into account small fluctuations of coupling strengths $h_k = H(x_k) - 1$. We consider local deviations of the phases from the synchronous cluster: $\psi_k = \varphi_k - \Phi$, and by substituting this in Eq. (1) obtain

$$\dot{\psi}_k = \sin\alpha + \frac{1}{N} \sum_{j=1}^{N} G(x_j - x_k) \sin(\psi_j - \psi_k - \alpha)$$

$$= \sin\alpha + \cos(\psi_k + \alpha) \frac{1}{N} \sum_{j=1}^{N} G(x_j - x_k) \sin\psi_j -$$

$$\sin(\psi_k + \alpha) \frac{1}{N} \sum_{j=1}^{N} G(x_j - x_k) \cos\psi_j.$$

Now we take into account that the deviations of the phases ψ_k are small and are nearly symmetrically distributed around zero. Thus, we assume

$$\frac{1}{N}\sum_{j=1}^{N} G(x_j - x_k)\sin\psi_j \approx 0,$$
$$\frac{1}{N}\sum_{j=1}^{N} G(x_j - x_k)\cos\psi_j \approx \frac{1}{N}\sum_{j=1}^{N} G(x_j - x_k) = H(x_k).$$

Therefore, we obtain the approximate dynamics of the phase deviation at site k as

$$\begin{aligned}\dot{\psi}_k &= \sin\alpha - H(x_k)\sin(\psi_k+\alpha)\\ &= \sin\alpha - (1+h_k)\sin(\psi_k+\alpha).\end{aligned}$$

According to this equation, the dynamics of the perturbed phase is a steady state (i.e., the oscillator k belongs to the synchronous cluster) if $1 + h_k > \sin\alpha$ or $1 + h_k < -\sin\alpha$. In our case, where h_k is small and α is close to $\pi/2$, only the first condition is relevant. If it is violated, i.e., if $h_k < h_c = \sin\alpha - 1$, then the oscillators k starts to rotate and the synchronous state disappears.

Thus, the probability that the synchronous state disappears (and this leads, as we have shown above, to the appearance of chimera), is the probability that at least at one site $h_k < h_c$. Therefore, we have to analyze the distribution of minima of the field $H(x)$ defined as

$$H(x) = \frac{1}{N}\sum_{i=1}^{N} G(x - x_i), \tag{5}$$

where x_i are random positions on the interval $0 \leqslant x < 1$ with uniform density $w(x) = 1$.

The statistics of the field H can be evaluated as follows. First, due to normalization $\int_0^1 G(x)dx = 1$, we get $\langle H\rangle = 1$. Next, using independence of positions x_i, it is straightforward to calculate the covariance of H (this calculation is completely analogous to a calculation of the correlation function of the shot noise (sequence of independent pulses, the Campbell's formula) [18]:

$$\begin{aligned}K(y) =& \langle H(x)H(x+y)\rangle - 1 = N^{-1}\frac{\kappa^2 B(\kappa,y)}{4\sinh^2\frac{\kappa}{2}},\\ B(\kappa,y) =& \frac{\cosh\kappa y}{2} + \frac{y[\cosh\kappa(y-1) - \cosh\kappa y]}{2}\\ &+ \frac{\sinh\kappa y - \sinh\kappa(y-1)}{\kappa}.\end{aligned} \tag{6}$$

One can see that the variance of field H decays as expected $\sim N^{-1}$. One can argue that for large N, as a sum of N statistically independent contributions, the field $H(x)$ is Gaussian, and this indeed is nicely confirmed by numerics (not shown). However, we are interested in the distribution of the *minima* of this field, and obtaining it is a nontrivial task, because of correlations Eqs. (6) (see Ref. [19]). These correlations, however, do not depend on N except for the overall factor N^{-1}, and therefore the distribution of the minima $h_{\min}$ on the lattice of size N scales like

$$\mathrm{Prob}(h_{\min} < \xi, N) = Q(\xi N^{1/2}),$$

where function Q is universal (for large N). The scaling of the random variable is $\sim N^{-1/2}$, because the variance scales $\sim N^{-1}$. Substituting here the threshold $h_c = \sin\alpha - 1$, we can express the probability to observe synchrony in a lattice of size N as

$$\begin{aligned}\mathrm{Prob}(syn, N) &= 1 - \mathrm{Prob}(chim, N)\\ &= 1 - \mathrm{Prob}(h_{\min} < h_c, N) = 1 - Q(h_c N^{1/2}).\end{aligned} \tag{7}$$

As mentioned above, we cannot derive an analytic expression for $Q(y)$, because one needs to find a distribution of minima among correlated Gaussian variables. However, it is straightforward to find this distribution numerically. If one determines the distribution $F_M(\xi)$ of minima of the field Eq. (5) in a lattice of size M, then according to the scaling relation one gets $Q(y) = F_M(yM^{-1/2})$.

In Fig. 2 we compare this estimate with direct numerical simulations, using three distributions F_M obtained for $M = 128,\ 256,\ 512$. These curves are practically indistinguishable, what is just another manifestation of the validity of the scaling $H_{\min} - 1 \sim N^{-1/2}$. The curve lies below the numerical data, what means that the adopted estimate is rather crude. Nevertheless, it correctly predicts that for $N \lesssim 1000$ practically all configurations lead to a chimera state.

IV. TRANSITION FROM SYNCHRONY TO CHIMERA FOR MOTILE PARTICLES

In this section we consider motile particles with random trajectories. In all cases reported in this section below, we start at $t = 0$ with particles regularly distributed on the ring, i.e., $x_i(0) = (i-1)/N$. The phases are set to be equal, so that the initial state is the perfectly synchronized one. Because of irregular motion, disorder in the positions of the particles appears. At rather large times the particles can be considered as noncorrelated, thus their positions are fully random on the ring. This, as we have seen in Sec. III, facilitates a transition to a chimera. Moreover, as in the course of time evolution different random configurations appear, eventually one which does not support synchrony will lead to a transition to a chimera (we illustrate this in Fig. 3). Thus, on the contrary to the case of static configurations of Sec. III, we expect that a transition from synchrony to chimera will always be observed even at system sizes as large as $N = 8192$.

In our simulations we used a criterion of the transition to a chimera based on the global order parameter R. We started with a synchronous state where $R = 1$. During a long transient period, the order parameter stays close to one, unless a large enough portion of the oscillators becomes asynchronous. Here the order parameter drops, and we adopted the moment of crossing the threshold $R = 0.85$ as a criterion of transition to chimera. Additionally, we checked the local order parameters—in all simulations after the transition the maximal (over the lattice) value of the local order parameter was very close to one, indicating for presence of a synchronous domain, and the minimal value of the local order parameter fluctuated around values 0.2–0.4, indicating for presence of a disordered domain. We illustrate this with Fig. 4. In this

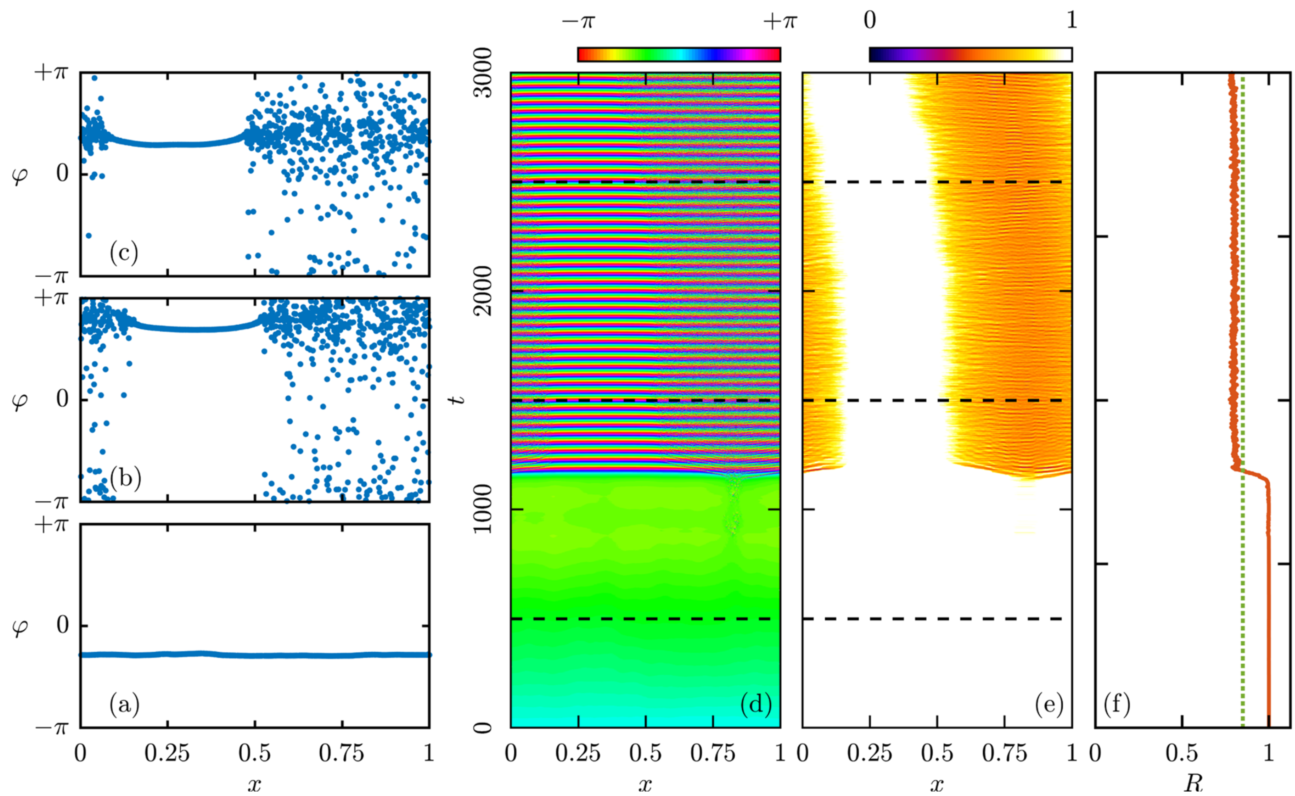

FIG. 3. The same as described in the caption of Fig. 1 but for diffusive particles with $\sigma = 10^{-3}$. Initially all the particles are placed equidistantly on the circle, and have equal phases. Developing at $t \approx 1000$ chimera pattern slowly moves along the circle, due to random rearrangement of particles positions. Panels (a, b, and c): snapshots of the phase distributions $\varphi(x, t)$ at (a) $t = 500$, (b) $t = 1500$, (c) $t = 2500$. Panel (d): the spatiotemporal dynamics of phases $\varphi(x, t)$. Panel (e): the absolute value of the local order parameter $Z(x, t)$. Panel (f): the dynamics of the global order parameter $R(t)$.

figure we show an overlap of 50 runs, shifted in time so that the crossing of the level $R = 0.85$ is at time zero. One can see that, although a reappearance of full synchrony is not excluded, practically it never happens (at least during the explored durations of the simulations, see discussion of potential timescales in Conclusion).

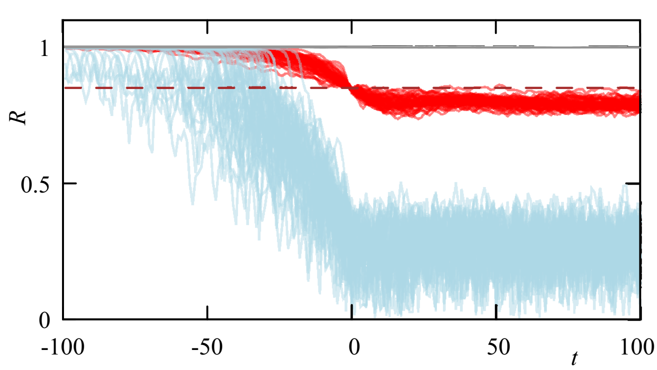

FIG. 4. Overlap of 50 independent runs of simulations for diffusive particles with $N = 1024$ and $\sigma = 0.003$. Red lines: evolution of the global order parameter. Time axes are shifted so that all red lines cross the adopted threshold $R_c = 0.85$ (dashed line) at $t = 0$. Light blue (gray) lines: minimal (over the lattice) values of the local order parameter (calculated over 17 neighboring sites). One can see that for positive times these values never exceed 0.6, thus proving existence of a disordered domain. Black lines : maximal values of the local order parameter; they are very close to one for all times (meaning that always a synchronous domain is present).

Our main interest below is the dependence of the transition time (from synchrony to chimera) on the parameters of noise and on the system size. In the system of differential equation (1), positions of the particles x_i can be considered as parameters. We start with a stable fixed point in this system, which does exist for regularly spread particles. Slow motion of particles means slow variation of the parameters in Eq. (1), and initially the stable steady state continues to exist. However, when the set of parameters reaches a bifurcation point (numerical experiments show that this is a saddle-node bifurcation, like in a disordered lattice [17]), the steady state disappears and another, chimera state, appears. Thus, what we want to study, is the time to bifurcation.

There is also another view on the transition to a chimera. In the starting configuration, where the oscillators are equidistantly distributed, the acting field $H(x)$ [see Eq. (5)] is constant. When the particles start moving, this field is no more constant, so one observes roughening of $H(x)$ [20]. This roughening continues until the minimum of the field becomes small enough to induce the bifurcation. This picture suggests that one can expect the average time of the transition $\langle T \rangle$ to scale with parameters of the problem: characteristic spread of random velocities of the particles and their number. We explore this idea of scaling below.

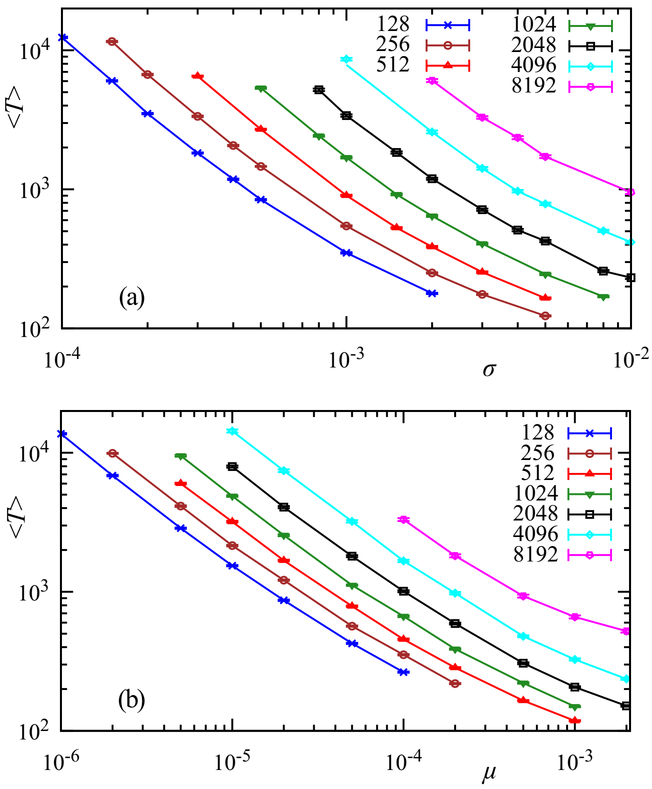

FIG. 5. Average time for a transition from synchrony to chimera for different N [from $N = 128$ (bottom curve) to $N = 8192$ (top curve), values of N increase by factor 2]. All data points are shown with error bars, which are smaller than the marker size. Panel (a): for diffusive motion of the particles [Eq. (3)] in dependence on the diffusion parameter σ. Panel (b): for ballistic particles with Gaussian distribution of velocities, in dependence on the standard deviation μ.

We consider two basic setups for the random motion of particles:

(1) **Diffusive motion.** Here we consider diffusive motion of the particles according to Eq. (3). The average transition times from synchrony to chimera are presented in Fig. 5(a). As expected, the time grows with the number of particles N, and for small diffusion rates σ.

(2) **Ballistic motion.** Here we assume that the particles move with constant velocities v_i, which are chosen from the normal distribution with standard deviation μ. The average transition times are shown in Fig. 5(b).

Figure 6 illustrates the distribution of the transition times T. It shows two examples, one for diffusive particles with $N = 1024$ and $\sigma = 0.002$, and another for ballistic particles with $N = 1024$ and $\mu = 2 \times 10^{-4}$. In both cases the distribution appears to be exponential, with an offset at small times.

Next, we discuss scaling properties of the time to chimera. We look for a scaling relation in the form

$$\langle T(c, N)\rangle = N^a f\left(\frac{c}{N^b}\right), \tag{8}$$

where c stands for one of the parameters μ, σ, and constants a, b generally depend on the setup. We, however, could not

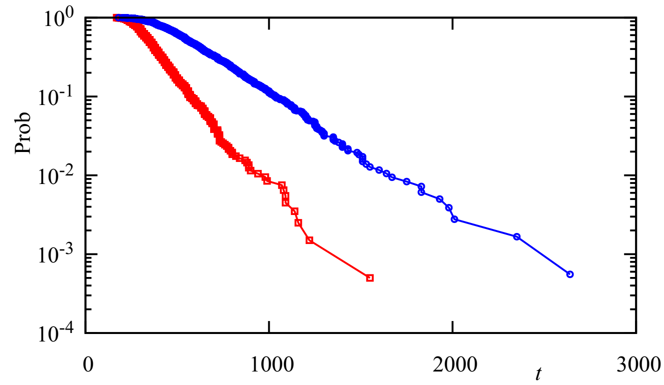

FIG. 6. Cumulative distribution of the transition times for the onset of chimera: the vertical axis displays probability that this time is larger than t. Blue circles: diffusive particles with $N = 1024$ and $\sigma = 0.002$; red squares: ballistic particles with $N = 1024$ and $\mu = 2 \times 10^{-4}$.

fit all the data according to a unique law [Eq. (8)]. As we illustrate in Fig. 7, taking data for the interval of system sizes $128 \leqslant N \leqslant 1024$ allows for achieving a very good collapse of data points using scaling in the form of Eq. (8), with $b = 0.45$ and $a = 0.15$ for both cases (diffusive and ballistic motions). However, using these parameters for larger system sizes $N \geqslant 2048$ does not lead to a good collapse of points. Rather we use for large N values $b = 0.3$ and $a = 0.6$ for the ballistic case and $b = 0.35$ and $a = 0.65$ for the diffusive case, but they result only in an approximate collapse of data points.

We attribute this absence of a universal scaling to the properties of the quenched randomness described in Sec. III. As it follows from Fig. 2, for $N \lesssim 1024$ it is enough for particles to achieve random independent positions on the circle, then the transition to chimera is nearly certain. In contradistinction, for larger populations there is a finite probability for a random quenched configuration to possess synchrony. This leads to an increase of the transition time: random motion of the particles explores different configurations, until one that does not posses synchrony is found and the transition to chimera occurs. This explains different scalings with a crossover near $N = 1024$. Moreover, we expect that the scaling observed for $2048 \leqslant N \leqslant 8192$ will not extend to larger system sizes, because according to Fig. 2, for such large systems, the probability of the transition in quenched configuration drastically reduces, so that the time to achieve a chimera will be extremely large, if not infinite.

V. CONCLUSION

In this paper we studied the effect of the oscillators position disorder on the chimera state in the Kuramoto–Battogtokh model of nonlocally coupled phase oscillators on a ring. The level of disorder is basically determined by the number of units N, it disappears in the thermodynamic limit $N \to \infty$. Our main finding is that large disorder facilitates stability of chimera, and for sizes of populations below some level, it is practically impossible to observe a stable synchronous regime in a setup with a quenched disorder. For slow random

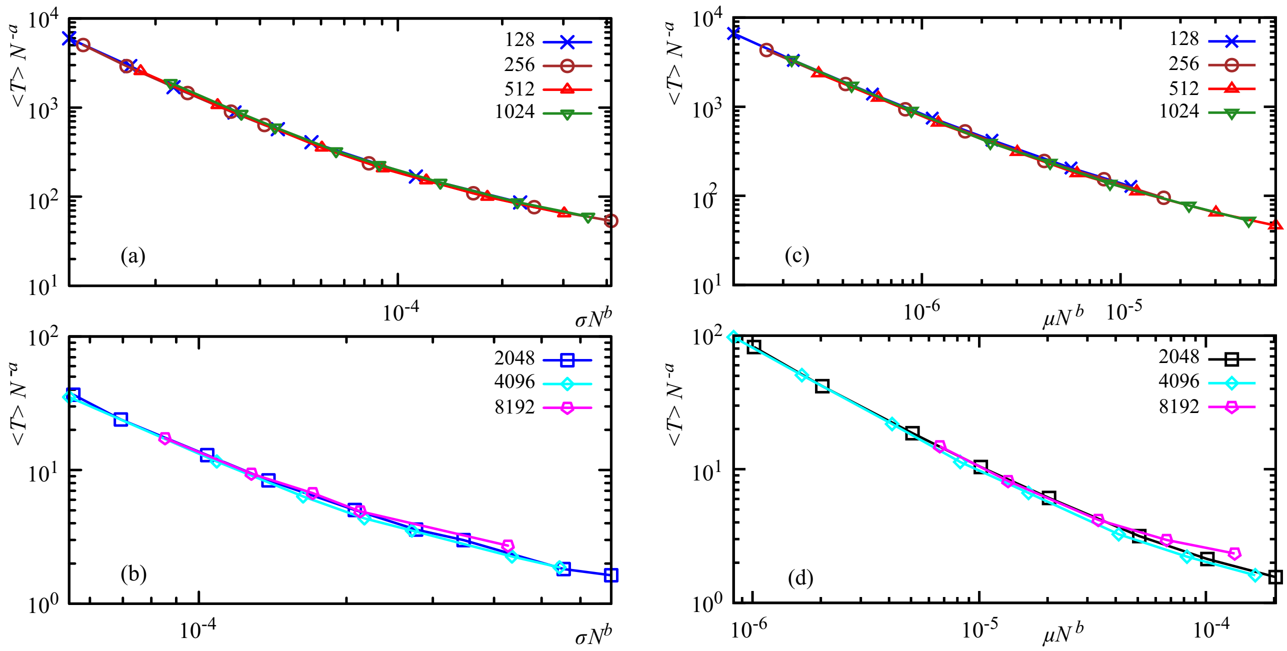

FIG. 7. The same data as in Fig. 5, but in scaled coordinates [diffusive particles in panels (a, b), ballistic particles in panels (c, d)]. Top row [panels (a, c)]: scaling for $128 \leqslant N \leqslant 1024$ with $b = 0.45$ and $a = 0.15$. Bottom row [panels (b, d)]: scaling for $2048 \leqslant N \leqslant 8192$ with $b = 0.35$ and $a = 0.65$ for diffusive particles, and $b = 0.3$ and $a = 0.6$ for ballistic particles.

motions of the particles, in the explored range of system sizes up to $N = 8192$, we observed a transition from synchronous initial configuration to a chimera in all realizations. Even when synchrony has a finite probability to exist in a quenched configuration, slow variations of positions of particles lead eventually to a configuration where synchrony state does not exist, so that a chimera develops.

We explored the scaling properties of the transition to chimera and found that for both diffusive and ballistic motions, the scaling exponents in the relation Eq. (8) are nearly the same. Due to a nontrivial dependence of the probability of the existence of synchrony already for a quenched disorder, the scaling is different for relatively small sizes N (where synchrony is practically never observed) and for larger sizes, where in the quenched case there is a finite probability for synchrony to survive. We, however, have not explored very large populations $N > 8192$, because of computational restrictions.

It is instructive to discuss a question, whether the observed chimera is a final state, or a synchronous state could reenter. This possibility is definitely excluded for the cases of quenched disorder, because in our simulations, started from a synchronous initial condition, appearance of chimera means absence of a stable synchronous solution. For moving particles, the situation is more subtle. Here it is not excluded that during the particles motion a configuration possessing a stable synchronous state appears and exists for some time interval. On the other hand, from studies [4] it is known that chimera in a homogeneous static lattice is a supertransient and after a large time interval (exponential in N), evolves into a synchronous state. Thus, potentially a synchronous state could re-emerge spontaneously. This would be, however, "doubly improbable," because one needs a superposition of the mentioned two extremely rare events. In our simulations we never observed re-entrance of synchrony.

We stress here that we studied the Kuramoto–Battogtokh model for the "standard" parameters κ, α used in Ref. [1]. The domain of existence of chimera and its basin of attraction may depend significantly on these parameters. Extension of the obtained results on other domains of parameters and on other setups where chimera patterns exist is a subject of ongoing study.

In this paper we focused on the regime of very slow motion of the particles, including the static (quenched) case. Preliminary simulations show that the regimes with fast particles can differ significantly, this is a subject of ongoing research. Another interesting case for future exploration is one close to the thermodynamic limit, where finite-size fluctuations are small. Here an analytical description based on the Ott–Antonsen reduction might be possible, to be reported elsewhere.

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