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Transition to synchrony in chiral active particles

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Abstract

PAPER

I study deterministic dynamics of chiral active particles in two dimensions. Particles are considered as discs interacting with elastic repulsive forces. An ensemble of particles, started from random initial conditions, demonstrates chaotic collisions resulting in their normal diffusion. This chaos is transient, as rather abruptly a synchronous collisionless state establishes. The life time of chaos grows exponentially with the number of particles. External forcing (periodic or chaotic) is shown to facilitate the synchronization transition.

1. Introduction

Active or self-propelled particles are a subject of active current research (see introductory review [1]). For microscopic particles, random interactions with an environment are essential, and one speaks about active Brownian particles, subject to noisy forces. For macroscopic particles, existence of random forces is not so obvious, but here one also often introduces them to model observed fluctuations in the motions of animals and birds (cf famous Vicsek model [2]). A large class of active particles constitute *chiral* active particles, natural trajectories of which are not straight lines, but circles. The origin of chirality can be asymmetry in the particle shape or in the propulsion mechanism; also external magnetic field may lead to circular motion (see discussion and examples in [1]).

There are different models for interaction of active particles. In many cases, inspired by the Vicsek model [2], one assumes an aligning interaction: neighboring particles 'prefer' to align their velocities. In the context of chiral Brownian (i.e. noise-driven) particles, such an aligning interaction can lead to an appearance of synchronized rotating clusters (see, e.g., references [3-6]). In this paper, I consider *deterministic* particles *without alignment forces*. The main finding is that this system demonstrates a transition from supertransient chaos to synchronous clusters.

I introduce the basic model in section 2. In section 3 I demonstrate that while for small times, circling colliding active particles demonstrate chaos leading to their diffusion, at large times a transition to a synchronous state without collisions occur. The life time of chaos grows exponentially with the number of particles, what allows to speak about supertransients, typical for spatio-temporal chaos. In section 4 I demonstrate that external periodic or random forcing induces a transition to a collisionless state already at small times. I conclude with discussion in section 5.

2. Model formulation

I consider active particles in two dimensions, with circular natural trajectories. The two components of velocity $(v = \dot{x}, u = \dot{y})$ for one particle obey following equations:

$$\dot{v} = -\omega u + \mu (W^2 - u^2 - v^2) + m^{-1} F_x,$$

$$\dot{u} = \omega v + \mu (W^2 - u^2 - v^2) + m^{-1} F_v.$$
 (1)

Here *W* is a steady speed, which a particle attends if the other forces $F_{x,y}$ vanish; parameter μ describes the rate of the relaxation to this steady speed; *m* is the particle's mass. The velocity field rotates with frequency ω . In a steady state, an isolated particle rotates on a circle of radius W/ω with frequency ω .

It is worth noting that equation (1) is widely used in synchronization and coupled oscillators studies as the Stuart-Landau model (see, e.g., [7]). In this interpretation two variables u, v describe a state of an autonomous oscillating system close to the Hopf bifurcation point. It is also well-known that under a periodic or random force this oscillator synchronizes [7], this effect will be explored in section 4.

Below I consider two types of forces acting on particles. The first force is the interaction between the particles. I assume, following reference [8], a conservative repulsing interaction governed by a truncated Lennard-Jones potential, which depends on the distance R between the particles:

$$V(R) = \begin{cases} \epsilon \left[\left(\frac{\sigma}{R}\right)^{12} - 2\left(\frac{\sigma}{R}\right)^6 + 1 \right] & \text{for } R \leqslant \sigma, \\ 0 & \text{for } R > \sigma. \end{cases}$$
(2)

This potential takes from the full Lennard-Jones potential only its repulsing part, and the attracting part is absent. Thus, this potential mimics not-so-hard discs with radius $\sigma/2$, which repulse each other when collide, and do not interact aside of collisions. I stress here that the interactions have no any alignment action (the latter is often assumed in models of Vicseck type). If there where no activity and chirality [i.e. $\omega = \mu = 0$ in (1)], then the model reduces to a Hamiltonian one of elastically colliding disks. In the case of hard disks it is believed that the dynamics is fully chaotic (the Boltzmann–Sinai hypothesis), although the proof [9] has some restrictions. For smooth potentials, stable periodic orbits in the Hamiltonian dynamics may appear [10]. Much less is known for colliding active particles, but our simulations in section 3 indicate for chaos.

In addition to interaction forces described by (2), I will consider external forces specified in section 4.

Below I study numerically an ensemble of active particles with circular orbits (1), (2) in a periodic geometry, i.e., on a torus $L \times L$. I fix $\mu = 2$, $W = \omega = 1$, $\sigma = 0.2$, m = 1, and $\epsilon = 0.1$ throughout the paper. The main parameters to explore is the number of the particles N and the density $\rho = NL^{-2}$. Numerical integration has been performed using the standard fourth order Runge–Kutta method with step $\Delta t = 2\pi/800$. In all cases the calculations start from a maximally random initial state: initial positions of the particles are randomly chosen from a uniform density on the square (of course, overlapping is avoided), initial velocities have the form $u = W \cos \alpha$, $v = W \sin \alpha$, where α is uniformly distributed on a circle $[0, 2\pi)$

3. Chaotic state and spontaneous transition to synchrony

3.1. Quasistationary chaotic state

Because collisions of hard disk have a scattering property with an essential degree of instability (like dispersive billiards), one can expect chaos in colliding active particle described by equations (1) and (2). Due to multiple collisions, velocity of a particle is random, so its motion is a diffusion in two dimensions. I illustrate this with figures 1 and 2. They show trajectories of N = 20 particles in a particular run. Up to time $T_{st} \approx 27\,000$ (I measure time in periods of rotations) motion of all particles is irregular.

Quantitatively, a good characteristics of irregularity is the mean diffusion constant of the particles. It can be calculated from the mean squared displacement after a large time interval

$$D = \frac{\langle (x(T) - x(0))^2 + (y(T) - y(0))^2 \rangle}{T}.$$

Diffusion in the system is normal, as the graph of $\langle (x(T) - x(0))^2 + (y(T) - y(0))^2 \rangle$ versus time interval *T* (figure 3(a)) shows. Because *D* is an intensive quantity, one can expect that it depends on the intensive parameter—density of particles ρ , but not on the number of particles *N*. However, calculations of the diffusion constant figure 3(b) show, that for a small number of particles a significant depletion of the diffusion constant is observed. I attribute this to correlations which appear in small communities where the same particles collide many times. In large ensembles, a particle has again and again new neighbors, so that correlations effectively disappear [see nearly coinciding values for N = 30 and N = 40 in figure 3(b)].

The diffusion constant non-monotonically depend on the density of the particles. Heuristically, this can be explained as follows. For large densities, the system is quite crowded, and the mean free rotation time between collisions is small. Thus, diffusion results from a sequence of uncorrelated small steps. For smaller density,







Figure 2. Trajectories of particles for the same data as figure 1. Positions of the particles are shown stroboscopically with period of rotation $2\pi/\omega$. Therefore the positions at the synchronous stage are fixed points in this two-dimensional representation, and thus not visible.

the free rotation time increases and the length of a step between two collisions becomes closer to its maximal value, diameter of the circles; thus diffusion becomes enhanced. With further decrease of density, collisions become more seldom while the characteristic displacement between collisions remains the same; thus diffusion constant decreases.

3.2. Spontaneous transition to synchrony

The main observation of this work is that the chaos described above is in fact a transient state, it evolves eventually into a configuration without collisions; such a transition can be seen at $T_{st} \approx 27\,000$ in figure 1. Indeed, a set of particles (1) has an absorbing synchronous state where all the velocities are equal: $v_1 = v_2 = \cdots = v_N$, $u_1 = u_2 = \cdots = u_N$. In this state the particles rotate synchronously, the distances between them remain



perfect straight lines confirming normal diffusion of particles. Panel (b): diffusion constants (obtained by averaging over 500 independent runs) for different N and ρ .

constant, and they do not collide. Thus this state, in which the particles do not interact, continues forever. Such a state can exist also in a Hamiltonian setup, but there it can occur only for specially constructed initial conditions. If collisions in a set of Hamiltonian discs occur, they cannot disappear, because of the reversibility of the dynamics. For active particles with a non-Hamiltonian dynamics, there is no such a restriction.

I stress that the state in figure 1 at $T > T_{st}$ is not completely synchronous, as the phases of rotation of different particles do not coincide. It is sufficient to achieve a state where collisions disappear, such a regime, which I call absorbing collisionless state, continues forever. In fact, this state always establishes in the numerical simulations performed in the given range of parameters, so one can attribute it as a globally attracting one.

I illustrate the collisionless synchronous state corresponding to $T > T_{st}$ of figure 1 in figure 4. Figure 1 shows that the transition to synchrony is quite abrupt, so this is an example of type-II supertransients according to classification of [11]. This allows one to speculate, that the mechanism of the transition is in exploration by the chaotic system of different possible configurations, until a configuration without collisions is 'found'.

The time at which a synchronous absorbing state appears depends on the initial configuration of particles, it is distributed according to an exponential law as figure 5 demonstrates. In figure 6 I show dependence of the mean time to achieve a collisionless state on the number of particles and on density ρ (the number of independent runs for the statistics in figures 5 and 6 was in the range from 700 to 4000). The main feature is that the life time of a chaotic state grows exponentially with the number of particles *N*. This growth is, however, perfectly exponential for large enough ensembles $N \ge 20$ only; for smaller system sizes the growth is slightly faster than exponential. The exponent appears to vary only weakly with the density parameter. Thus, this system belongs to a class of extended systems with chaotic supertransients [12, 13]. Figure 6 also shows that the law of exponential growth with the number of particles only weakly depends on the density parameter.





4. Driven transition to synchrony

Noninteracting particles are described by a set of effective Stuart–Landau oscillators (1). Thus, such an ensemble can be synchronized by an external periodic or random force, as described in the theory of synchronization [7]. One can expect that the same happens also in the presence of the collisions, although the synchronization onset can be retarded due to them. I Illustrate the effect of a periodic force $F_x = \gamma \cos(\omega t)$ on the ensemble in figure 7. A small force has almost no effect, the mean life time of the chaotic state is almost the same as for autonomous particles. At large values of γ , the transition occurs within a few periods, the mean life time is almost the same for different *N* and ρ . Noteworthy, at intermediate force amplitudes $0.01 < \gamma < 0.03$, the reduction of the mean life time of chaos is mostly pronounced for systems with low density (see curves with $\rho = 1$ for N = 20 and N = 30).

Common noise is another source of synchrony in ensembles of uncoupled oscillators [7]. Here I report on numerical experiments where noise was in the form of a Poissonian sequence of delta-pulses:

$$F_x = \sum_n a_n \delta(t - t_n),\tag{3}$$







where t_n are Poissonian time events appearing with rate τ^{-1} , and a_n are independent amplitudes of the pulses taken from a Gaussian distribution with standard deviation σ . Mean life times of chaos are shown in figure 8. Remarkably, under noisy force the mean life time of chaos is nearly a constant, it does not depend on the number of particles and on the density. This can be attributed to the fact that the force is rather strong in our setup. As a result, the noisy driving not simply 'assists' natural transition to synchrony, but rather destroys it. The transition time is therefore determined by the time of synchronization by noise of individual, noncolliding particles. So, for small number of particles common noise retards transition to synchrony compared to the spontaneous one. For large number of particles the effect is opposite, here for N = 30 noise reduces the life time of chaos by a factor larger than 20. A very weak dependence on the mean life time of chaos on the number of particles and on the density allows for representing the dependence of this time on the parameters of the forcing σ , τ by averaging over N, ρ . The results presented in the inset of figure 8 show that this time grows with γ and τ (in the explored range $0.1 \leq \sigma \tau \leq 0.3$).





5. Conclusion

In this paper I studied an ensemble of active particles with circular natural trajectories, subject to elastic collisions. This setup differs from previous studies in two aspects: (i) the basic dynamics is purely deterministic (except for the special considered case of a common random force), and (ii) there are no aligning interactions. I show that the autonomous system demonstrates supertransient chaos. Starting with random initial conditions, for a long interval of time, chaotic dynamics is observed, leading to a normal diffusion of the particles. However, the final state is a regime without collisions, where neighboring particles effectively synchronize their circular rotations. I demonstrate that the life time of chaos depends on the density of particles, and grows exponentially with the number of them. Furthermore, I show that the transition to synchronous rotations can be induced by an external periodic force, common for all particles (this effect has been previously reported in other context in references [14, 15]). Also I demonstrate that a random common force also leads to a synchronous dynamics without collisions. Forced transitions have a certain transient time which for large forces does not depend on the number of particles. In this paper we considered an ideal case of identical particles. Preliminary calculations show that spontaneous regularization of the dynamics happens also for inhomogeneous populations (e.g., for different frequencies of the particles); more details will be presented elsewhere.

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Data availability statement

The data that support the findings of this study are available upon reasonable request from the authors.

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