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Synchronization, zero-resistance states and rotating Wigner crystal

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Abstract. We show, in a framework of a classical nonequilibrium model, that rotational angles of electrons moving in two dimensions (2D) in a perpendicular magnetic field can be synchronized by an external microwave field whose frequency is close to the Larmor frequency. The synchronization eliminates collisions between electrons and thus creates a regime with zero diffusion corresponding to the zero-resistance states observed in experiments with high mobility 2D electron gas (2DEG). For long range Coulomb interactions electrons form a rotating hexagonal Wigner crystal. Possible relevance of this effect of synchronization-induced self-assembly for planetary rings is discussed.

PACS. 73.40.-c Electronic transport in interface structures – 05.45.Xt Synchronization; coupled oscillators – 05.20.-y Classical statistical mechanics

The discovery of microwave-induced resistance oscillations (MIRO) [1] and of striking zero-resistance states (ZRS) of a 2DEG in a magnetic field [2,3] attracted a great interest of the community. A variety of theoretical explanations has been pushed forward to explain the appearance of ZRS (see Refs. in [4]). Many of these approaches provide certain MIRO which at large microwave power even produce a current inversion. Although there are arguments in the literature that ZRS are created as a result of some additional instabilities which may compensate currents to zero, the understanding of underlying mechanisms is missing. Hence, a physical origin of ZRS still remains a puzzling, challenging problem.

In this work we suggest a generic *classical* physical mechanism which leads to a suppression of electron-electron collisions and creates ZRS. Its main element is the synchronization phenomenon which has abundant manifestations in science, nature, engineering and social life [5,6]. A simple picture of the effect is the following: a microwave field excites electrons and switches on dissipation processes in energy, which compensate microwave-induced energy growth, thus creating a nonequilibrium steady-state distribution. Due to this dissipation, when the microwave frequency ω is close to a resonance with the Larmor frequency ω_B, the synchronization of the phases of Larmor rotations of electrons with the phase of microwave field is established. In this way all electrons start to oscillate coherently: each of them is locked by the external force and therefore they are in phase with each other. This coherent dynamics is typical for synchronization in ensembles of nonequilibrium oscillators; quite understood physical examples are laser arrays and networks of Josephson junctions, but one observes such a synchronization also in non-physical systems like populations of blinking fireflies [5,6], pedestrians on a bridge [7], and applauding audience [8]. But compared to other oscillators, the synchronization of moving electrons brings a new element not presented in the common synchronization studies: due to synchrony the collisions between electrons disappear. This leads to a drastic drop of the collision-induced diffusion constant D and to creation of ZRS. We note that the diffusion D is proportional to experimentally measured resistance R_{xx} since $R_{xy} \gg R_{xx}$ [2,3] and hence, $D \propto \sigma_{xx} \propto R_{xx}/R_{xy}^2$. Here R_{xx}, R_{xy} are resistance tensor components along and perpendicular to an external weak static electric field applied along x-axis. Also σ_{xx} is a conductivity tensor component along x-axis (the conductivity tensor is given by the inversion of resistivity tensor). A simple image of such synchronized electrons is given by an ensemble of particles randomly distributed on a 2D plane, which rotates as a whole – because all particles rotate in phase – on a Larmor circle of radius $r_B = v_F/\omega_B$ with frequency ω_B. Indeed, in such a rotating ensemble particles never collide, and we demonstrate below that this can happen with 2D electrons synchronized with a microwave field phase in a magnetic field B. The synchronization origin of ZRS allows one also to understand qualitatively why ZRS exist only in high mobility samples. Indeed, it is well known that synchronization remains robust to a weak noise but disappears at strong one [5], hence a weak

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impurity scattering will not destroy ZRS. It is also important to note that the above picture is based on a classical nonequilibrium dynamics. Such a classical approach is justified since in the experiments [2,3] the Landau quantum level is rather large $n_L \sim 100$. Thus, we start our analysis with a classical mechanics treatment and will turn to a discussion of quantum effects later.

To justify the synchronization picture of ZRS described above we perform extensive numerical simulations using two main models of classical electrons (particles) with short range and Coulomb interactions. In the simplest setup, we model particle dynamics with short range interactions in magnetic and microwave fields with the Nosè-Hoover (NH) thermostat (see e.g. [9,10]) combined with interactions treated in the frame of the mesoscopic multi-particle collision model (MMPCM) [11]. The NH thermostat produces an effective friction γ which keeps the average kinetic energy $\langle \mathbf{p}^2/2m \rangle$ equal to a given thermostat temperature T and equilibrates heating induced by a microwave field $\mathbf{f}_{ac} = \mathbf{f}\cos\omega t$. At the same time collisions in the MMPCM drive a system to an ergodic state with the equilibrium Maxwell distribution at a given temperature. In this way the particle dynamics is described by the equations:

$$\dot{\mathbf{q}}_i = \mathbf{p}_i/m \ , \ \dot{\mathbf{p}}_i = \mathbf{F}_i + \mathbf{f}_{Li} + \mathbf{f}_{ac} - \gamma \mathbf{p} \ , \quad (1)$$

$$\dot{\gamma} = [\langle \mathbf{p}^2 \rangle/(2mT) - 1]/\tau^2 \quad (2)$$

where $\mathbf{q}_i, \mathbf{p}_i$ are the coordinate and the momentum of particle i, $\mathbf{f}_{Li} = e[\mathbf{p}_i \times \mathbf{B}]/mc$ is the Lorentz force, $\mathbf{F}_i$ is an effective force produced by particles collisions, τ is the relaxation time in the NH thermostat and $\langle \mathbf{p}^2 \rangle$ means average over all N particles. We usually consider the case of a linearly polarized microwave field $\mathbf{f}_{ac}$ since numerical data give no significant dependence on polarization. In numerical simulations N particles are placed randomly on a square cell $L \times L$ which is periodically continued all over the plane. The collisions are treated in the MMPCM formalism, namely the main cell is divided into N_c small collision cells in which after a time step Δt the velocities of particles are reshuffled randomly but keeping conserved the momentum and energy of particles in the collision cell [11]. In absence of microwave radiation the system evolves to a usual thermal equilibrium with the Maxwell distribution. The average rate D of particles diffusion in space is computed via their displacements after a large time interval t. In presence of the microwave field the diffusion rate D is drastically changed in the vicinity of the resonance $\omega_B \approx \omega$ as it is shown in Figure 1 for typical values of parameters.

Figure 1 clearly shows the existence of a synchronization Arnold tongue inside which the diffusion drops to zero (here as well as in numerical simulations below, its residual numerical value $D/D_0 \lesssim 10^{-8}$ is essentially determined by roundoff errors and fit accuracy and is non distinguishable from zero). According to Figure 1 the synchronization regime and the ZRS exist inside the detuning range

$$|\omega_B - \omega| \leq sf/(mv_T) \ , \quad (3)$$

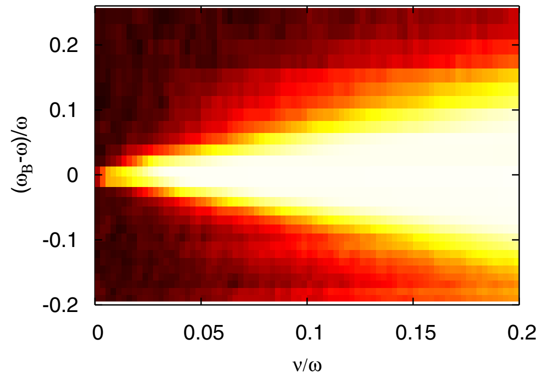

Fig. 1. (Color online) Color density plot of the normalized diffusion rate D/D_0 as a function of frequency detuning $(\omega_B - \omega)/\omega$ and rescaled microwave field strength ν/ω with $\nu = f/mv_T$ where $v_T = \sqrt{2T/m}$ is the thermal velocity and D_0 is the diffusion rate in absence of microwave at $\omega_B = \omega$. The system parameters are: $N = 1000$ $N_c = 4 \times 10^4$, $\omega\Delta t = 0.2$, $\omega\tau = 10$, $\omega t = 500$, $L/r_B = 10$, $D_0/D_c = 0.12$ (with $D_c = v_T^2/\omega$, $\rho = N/L^2$ and r_B taken at $\omega_B = \omega$, thus a number of particles inside a Larmor circle is $N_B = \pi r_B^2 \rho = \pi\rho v_T^2/\omega^2 = 10\pi$, $\omega = const$). Color intensity is proportional to D/D_0 (black for maximum $D/D_0 \approx 1.2$ and white for minimum $D/D_0 = 0$).

where $v_T = \sqrt{2T/m}$ and a numerical constant $s \approx 0.7$. We note that s is not sensitive to the relaxation time τ which has been varied by an order of magnitude. In fact the domain of ZRS given by (3) is very similar to a usual synchronization domain for one particle [5] which is also not sensitive to the dissipation rate. The origin of this similarity is rather clear: the synchronization with the microwave field phase eliminates collisions between particles, so that they move independently and hence the Arnold tongue becomes the same as for the synchronization of one particle by the periodic forcing. The fact that in the ZRS the collisions are eliminated, is confirmed by direct counting of the number of collisions in the numerical code and by computation of the synchronization parameter $S = \sum_{i<j} (\mathbf{v}_i - \mathbf{v}_j)^2/(N^2 v_T^2/2)$ which in the ZRS drops down to $S \sim 10^{-10}$ being determined by roundoff errors. This means that all particles have the Larmor phase synchronized with the microwave field phase while their positions in the coordinate space are disordered. Outside of the ZRS particles continue to diffuse with a rate D which is comparable with the unperturbed rate D_0. At small values of N_c and Δt when the collision rate becomes rather large and $D_0 \sim D_c = v_T^2/\omega \approx v_T r_B$, the ZRS regime is destroyed. Another model of collisions, in which the velocities of colliding particles are changed randomly in a bounded relatively small scattering angle, gives essentially the same result (3) for the ZRS.

To check the existence of the ZRS in the case of long range Coulomb interactions we use the molecular dynamics (MD) simulations of a classical two-dimensional electron liquid as described in [12]. The results obtained in [12] show that such an approach correctly describes plasmon modes in presence of a magnetic field even when

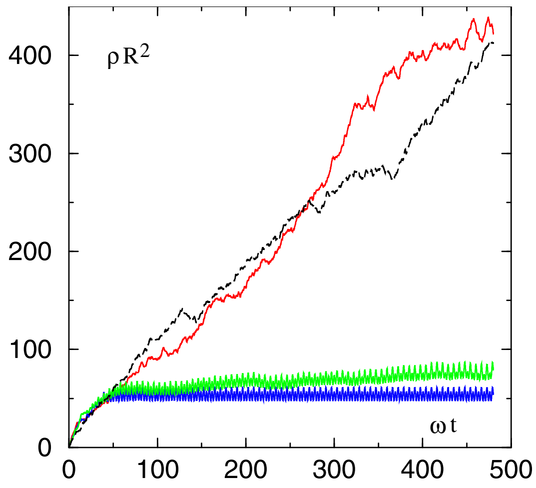

Fig. 2. (Color online) Dependence of electron square displacement R^2, rescaled by electron density ρ, on the rescaled time ωt. Here the Larmor frequency is $\omega_B = \omega$ at microwave field strength $f = 0$ (red top curve); $f/(mv_F\omega) = 0.059$ ($fa/E_F = 0.02$) for $\omega_B = \omega$ (blue bottom curve), $\omega_B = 0.875\omega$ (second from top black dashed curve), and $\omega_B = \omega$ with impurity scattering mean free path $l_i = 96 r_B$ (second from bottom green curve). Total number of electrons is $N = 100$ and $N_B = \pi\rho v_F^2/\omega^2 = 34.7$. The linear fit gives the diffusion rates $D/D_c = 0.089, 0.068, 0.0040, 9\times 10^{-6}$ with $D_c = v_F^2/\omega$ (respectively for curves from top to bottom ordered at $\omega t = 400$).

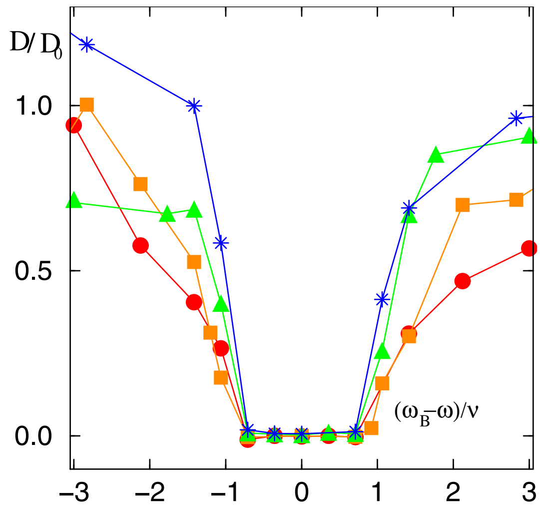

Fig. 3. (Color online) Dependence of rescaled diffusion rate D/D_0 on the rescaled frequency difference $(\omega_B - \omega)/\nu$. Here $\nu = f/mv_F$, D_0 is diffusion rate in absence of microwave at $\omega_B = \omega$, $fa/E_F = 0.02$ and number of electrons in a Larmor circle is $N_B = 2$ (stars), 8 (triangles), 34.7 (squares), 138.8 (points) with $D_0/D_c = 0.054, 0.089, 0.12, 0.14$ and $D_0/v_F a = 0.20, 0.35, 0.53, 0.64$ respectively. Total number of electrons is $N = 100$, $L = \sqrt{N/\rho} \approx 17.72\ a$.

the Coulomb energy $E_C = e^2/a$ is large compared to classical temperature T. Here, $a = 1/\sqrt{\pi\rho}$ is an average distance between electrons determined by the electron density ρ. We ensured that our numerical code with the Ewald resummation technique reproduces correctly the results presented in [12]. To equilibrate the heating induced by the microwave field we introduce in equation (1) an energy-dependent dissipation with $\gamma = \gamma_0(E - E_F)/E_F$ for $E = p^2/2m > E_F$ and $\gamma = 0$ for $E < E_F$. In such a way the dynamics remains Hamiltonian for $E < E_F$ while above E_F the dissipative processes are switched on as it is usually the case for 2DEG; thus E_F plays a role of Fermi energy [13]. Usually we use $E_F/T \approx 2$ but the obtained results are not sensitive to this ratio. The main part of simulations is done at an intermediate interaction strength $r_s = E_C/E_F = 0.3$ but we ensured that an increase(decrease) of r_s by a factor 7(3) does not change qualitatively the results (samples studied in [2,3] have $r_s \approx 2$). Also a variation of the dissipation rate γ_0 by an order of magnitude does not affect significantly the results and we present data at $\gamma_0 \approx 0.7 v_F/a$. The same is true for the total number of electrons varied from 20 to 200 at $\rho = const$, thus we present data at $N = 100$.

A typical example of the dependence of average electron square displacement R^2 on time is shown in Figure 2. The introduction of microwave field leads to the synchronization of electron Larmor phases and to a drastic drop of diffusion rate at $\omega_B = \omega$, formally by 4 orders of magnitude; the synchronization parameter S drops down to $S \approx 10^{-11}$ in this case that means that collisions are completely switched off. A shift in the Larmor frequency $\omega_B = 0.875\ \omega$ destroys synchronization and diffusion D is restored being close to its unperturbed value D_0 at $f = 0$. An introduction of a weak noise linked to impurity scattering with a scattering time t_i and mean free path $l_i = v_F\tau_i$ leads to a finite diffusion rate D which is however much smaller than D_0 until $l_i \gg r_B$ (see Fig. 2). A decrease of the mean free path down to $l_i \approx 10\ r_B$ destroys synchronization and restores a diffusion with rate $D \approx D_0$. We note that $l_i \approx 100\ r_B$ approximately corresponds to experimental conditions in [2,3].

The dependence of D on the frequency detuning is shown in Figure 3. The numerical data for Coulomb interactions between electrons show that the ZRS exist inside the synchronization window near the resonance $\omega_B \approx \omega$ with the width given by equation (3) where v_T should be replaced by v_F and $s \approx 0.8$. Inside the ZRS the diffusion drops practically to zero as discussed above. The validity of the relation (3) shows that the effect is not very sensitive to the type of interactions between particles.

However, the long range nature of Coulomb interactions significantly modifies the structure of the ZRS configuration: for short range interactions particles are distributed over the plane in a disordered way, while for the Coulomb interactions electrons form a hexagonal Wigner crystal as it is shown in Figure 4. The whole crystal (as well as each electron) is rotating in the plane with the frequency $\omega \approx \omega_B$ and rotation radius $r_B = v_F/\omega_B$. A remarkable property of the rotating Wigner crystal is that formally it is formed at a rather small parameter $r_s \approx 0.3$ while the usual Wigner crystal requires r_s values by more than two orders of magnitude larger [14]. We attribute this to synchronization of electron Larmor phases with the microwave field phase, what eliminates collisions between

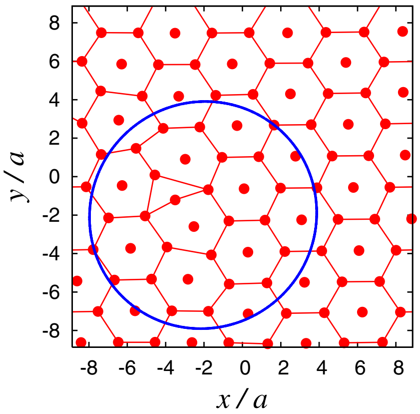

Fig. 4. (Color online) Instant image of the rotating Wigner crystal formed by $N = 100$ electrons (points) in a periodic cell with $L = \sqrt{N/\rho} \approx 17.72a$, $\omega t = 480$, $\omega_B = \omega$, $fa/E_F = 0.02$ and $N_B = 34.7$ (as in Fig. 2, bottom curve); the circle shows an orbit of one electron for $240 \le \omega t \le 480$; lines are drawn to adapt an eye showing a hexagonal crystal with a defect.

electrons and suppresses fluctuations, thus yielding an effectively large r_s in the rotating frame and allowing for a synchronization-induced self-assembly of 2DEG. In the crystal all Coulomb forces acting on an electron are compensated, thus the size of synchronization domain in frequency range given by equation (3) is essentially the same as for one-particle synchronization and is practically independent of dissipation rate γ_0 [5]. It is interesting to note that this result is rather different from the case of Kuramoto oscillators (see e.g. [5]) where a width of synchronization domain is proportional to a number of coupled oscillators. This difference should be attributed to importance of space dynamics of electrons in our case while this element is absent in the Kuramoto model. Thus in our case the interaction between particles in the synchronized regime is effectively compensated that is not the case for the Kuramoto model. Also in the ZRS system the dissipation appears only for electrons exited by a microwave field above the Fermi energy while in the Kuramoto model a dissipation is always present.

In conclusion, we have suggested a generic mechanism which for nonequilibrium classical rotational dynamics of an ensemble of particles in two dimensions produces synchronization of rotational angles of all particles with the phase of external driving periodic field. As a result, a rotating Wigner crystal is created and a collisional diffusion is suppressed by several orders of magnitude. We propose this effect as a possible mechanism of ZRS in 2DEG observed in [2,3]. In particular, it provides a realistic estimation of the microwave field at which these states appear. According to eqution (3) the relative size of ZRS plateau is $\Delta\omega/\omega \approx 2\nu/\omega \approx fv_F/\omega E_F$ that for experiments [2,3] with $E_F \sim 100$ K°, $v_F \sim 3\times10^7$ cm/s and $\omega/2\pi = 35$ GHz gives $\Delta\omega/\omega \approx 0.1$ if the field strength acting on an electron is $f/e \approx 5$ V/cm. This relative width is in a reasonable agreement with the experimental results [2,3,15] where unfortunately an exact value of f is not known. The synchronization energy scale $E_S \sim fr_B \sim 10$ K and the crystal Coulomb energy $E_C \sim 200$ K might be the origin of large energy scale $E_A \sim 10$ K in the ZRS activated transport with a temperature dependence $R_{xx} \propto \exp(-E_A/T)$ inside the ZRS [2,3]. The coherent rotation of electrons in the crystal creates a rotating current in 2D plane which in its turn generates a magnetic field $B_W \sim \mu_0 ev_F\rho \sim 1$ G parallel to 2DEG and rotating in the plane with a frequency close to ω. Another characteristic feature of this magnetic field B_W is that it exists only inside the ZRS where the synchronization condition (3) is fulfilled. Outside of synchronization domain the rotational phases of electrons are random and thus B_W becomes zero. Such a rotating magnetic field B_W is sufficiently strong and it can be detected experimentally inside the ZRS domain. We note also, that the collective crystal structure appearing due to synchronization can suppress the diffusion induced by impurities.

Although providing a ZRS, the simplified theory above fails to reproduce several important features observed in experiments. An important discrepancy from the experiments is that our theory gives synchronization only near the main resonance $\omega_B/\omega \approx 1$ while in the experiments ZRS exist also near integer low resonances j with $\omega_B/\omega \approx 1/j$. In the classical synchronization picture such resonances (higher-order Arnold tongues) are possible, however for nearly sinusoidal oscillations (rotations) and a sinusoidal force they are extremely narrow and we did not observed them in our numerical simulations. These higher-order resonances may appear if the forcing has higher harmonics or the rotations are highly nonuniform. Mechanisms for such nonlinearities certainly deserve a further investigation. One possibility might by the effect of additional spatial modulation of the underlying potential that may appear during molecular epitaxial growth of the samples [16]; this modulation may also produce a frequency shift in the rotational frequency that may be responsible for a resonance shift of the ZRS domain compared to the Larmor resonance (see [2,3,15]). Another feature that does not appear in our theory is the fine structure of the resistance vs. magnetic field dependence: outside of ZRS regions one observes an increase of the resistance R_{xx} by a factor 3 compared to the case without microwave field while our numerical data give an increase only by a factor 1.2 (see Fig. 3).

Our theory is based on the classical, nonequilibrium dynamics and it is crucial to analyze the relevance of quantum effects. In principle it is known that at small effective values of Planck's constant $\hbar_{eff}$ the synchronization is preserved while at large values $\hbar_{eff}$ it is destroyed by quantum fluctuations [17]. For 2DEG $\hbar_{eff} \sim 1/n_L$ and at $n_L \sim 100$ it is natural to expect that the synchronization is robust against quantum fluctuations. However, a reduction of n_L by an order of magnitude due to an increase of ω to a THz range may significantly enhance quantum noise and destroy ZRS. Further theory development is required to study quantum effects properly. The most important question is about the amount of electrons which are involved in the rotating Wigner crystal. Indeed, our classical studies show that all electrons are involved in this state but in the quantum case it is rather possible that only a finite fraction of electrons near the Fermi level contributes to the rotating crystal, while all other electrons will stay as a non-interacting background.

Finally we mention a striking analogy between the 2D electron gas in a magnetic field and a system of completely different spatial and temporal scales – planetary rings. Planetary rings are also essentially two-dimensional ensembles of particles, temperature there is very low and in the rotational frame the 2D dynamics of particles is similar to motion of electrons in a magnetic field [18]. A periodic force on the particles may be due to the effect of large moons. The resulting synchronous coherent state similar to described above may be responsible for enormously long life time ($\sim 10^{12}$ rotations) and sharp edges of planetary rings (e.g. $\sim$10 m for Saturn) [18,19].

It is also possible that the effects analyzed in this paper can be relevant to collective oscillations of ion and electron clouds in a Paul trap where a certain number of resonances with a driving microwave frequency has been observed [20,21] and a formation of small Coulomb crystals has been demonstrated [22]. We think that the mechanism discussed here can be viewed as a generic mechanism of synchronization-induced self-assembly of systems oscillating in space and its further investigation seems to be very promising.

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