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Phase synchronization of chaotic oscillators and analysis of bivariate data

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1. – Introduction

Synchronization phenomena in chaotic systems are a subject of intensive investigations. Mostly often the synchronization of mutually coupled oscillators is understood as coincidence of their states, $\mathbf{x}_1(t) = \mathbf{x}_2(t)$ [1-4]; it appeares only if interacting subsystems are identical. Otherwise, if the parameters of oscillators slightly mismatch, the states are close $|\mathbf{x}_1(t) - \mathbf{x}_2(t)| \approx 0$ but remain different [2, 5]. Other definitions imply the overlapping of power spectra of certain observables in the interacting systems [6, 7] or the coincidence of the attractor dimension of the whole system $(\mathbf{x}_1, \mathbf{x}_2)$ and partial dimensions (computed in the phase spaces spanned by $(\mathbf{x}_1)$ or $(\mathbf{x}_2)$) [8, 9]. These effects occur for a relatively strong forcing.

In this paper we describe the effect of *phase synchronization* of chaotic systems. This phenomenon results from rather weak interaction between two [10] or large number of oscillators [11, 12]; external synchronization of one oscillator by periodic or noisy force is described in [13]. Phase synchronization of chaotic systems is mostly close to synchronization of periodic oscillators. We define it as the occurrence of a certain relation between the phases of interacting systems (or between the phase of a system and that of an external force), while the amplitudes can remain chaotic and are, in general, uncorrelated. Of course, the very notion of phase and amplitude of chaotic systems is rather non-trivial; it is discussed in detail below.

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Analysis of phase relationship between two signals, naturally arising in the context of phase synchronization, can be used to approach a general problem in time series analysis. Namely, bivariate data are often encountered in the study of real systems, and the usual aim of the analysis of these data is to find out whether two signals are dependent or not. As the experimental data are very often non-stationary, the traditional techniques such as cross-spectrum and cross-correlation analysis [14] or non-linear characteristics like generalized mutual information [15] have their limitations. On the other side, sometimes additional physical assumptions can be made, that the observed data originate from two weakly interacting systems with slowly varying parameters. If the signals are close to periodic ones, then the usual approach is to consider them as an output of two coupled oscillators and to quantify their interaction by measuring the time-dependent phase difference between these signals. Here we demonstrate that this approach can be extended to the case of chaotic signals as well. In this case the phase difference can be effectively obtained from a bivariate time series by means of the analytical-signal approach based on the Hilbert transform [14].

2. – Instantaneous phase of signals and systems

A consistent way to define the phase of an *arbitrary* signal is known in signal processing as analytic-signal concept [14, 16, 17]. This general approach, based on the Hilbert transform and originally introduced in [18], unambiguously gives the *instantaneous phase and amplitude* for a signal $s(t)$ via construction of the *analytic signal* $\zeta(t)$, which is a complex function of time defined as

$$\zeta(t) = s(t) + j\tilde{s}(t) = A(t)e^{j\phi(t)}\,, \tag{1}$$

where the function $\tilde{s}(t)$ is the Hilbert transform of $s(t)$

$$\tilde{s}(t) = \pi^{-1}\mathrm{PV}\int_{-\infty}^{\infty}\frac{s(\tau)}{t-\tau}\mathrm{d}\tau \tag{2}$$

and PV means that the integral is taken in the sense of the Cauchy principal value. The instantaneous amplitude $A(t)$ and the instantaneous phase $\phi(t)$ of the signal $s(t)$ are thus uniquely defined from (1).

As one can see from (2), the Hilbert transform $\tilde{s}(t)$ of $s(t)$ can be considered as the convolution of the functions $s(t)$ and $1/\pi t$. Due to the properties of convolution, the Fourier transform $\tilde{S}(\omega)$ of $\tilde{s}(t)$ is the product of the Fourier transforms of $s(t)$ and $1/\pi t$. For physically relevant frequencies $\omega > 0$, $\tilde{S}(\omega) = -jS(\omega)$. This means that the Hilbert transform can be realized by an ideal filter whose amplitude response is unity, and phase response is a constant $\pi/2$ lag at all frequencies [14].

A harmonic oscillation $s(t) = A\cos\omega t$ is often represented in the complex notation as $A\cos\omega t + jA\sin\omega t$. It means that the real oscillation is complemented by the imaginary part which is delayed in phase by $\pi/2$, that is related to $s(t)$ by the Hilbert transform. The analytic signal is the direct and natural extension of this technique, as the

Hilbert transform performs the $-\pi/2$ phase shift for every frequency component of an arbitrary signal.

An important advantage of the analytic-signal approach is that the phase can be easily obtained from experimentally measured scalar time series. Numerically, this can be done via convolution of the experimental data with a pre-computed characteristic of the filter (Hilbert transformer) [16, 17, 19]. Although Hilbert transform requires computation on the infinite time scale, *i.e.* Hilbert transformer is an infinite impulse response filter, the acceptable precision of about 1% can be obtained with the 256 point filter characteristic. The sampling rate must be chosen in order to have at least 20 points per average period of oscillation. In the process of computation of the convolution $L/2$ points are lost at both ends of the time series, where L is the length of the transformer.

Although the analytic-signal approach provides the unique definition of the phase of *a signal*, we cannot avoid ambiguity defining the phase for a *dynamical system*, as the result depends on the choice of the observable. To approach this problem we remind that in the case of periodic oscillations the dynamics of a phase point on the limit cycle can be represented as

$$\frac{\mathrm{d}\phi}{\mathrm{d}t} = \omega_0 \,, \tag{3}$$

where $\omega_0 = 2\pi/T_0$, and T_0 is the period of the oscillation. It is important that starting from any monotonically growing variable θ on the limit cycle, one can introduce the phase satisfying eq. (3). Indeed, an arbitrary θ obeys $\dot{\theta} = \nu(\theta)$ with a periodic $\nu(\theta+2\pi) = \nu(\theta)$. A change of variables $\phi = \omega_0 \int_0^\theta [\nu(\theta)]^{-1}\mathrm{d}\theta$ gives the correct phase, where the frequency ω_0 is defined from the condition $2\pi = \omega_0 \int_0^{2\pi} [\nu(\theta)]^{-1}\mathrm{d}\theta$. A similar approach leads to correct angle-action variables in Hamiltonian mechanics. From (3) it is evident that the phase corresponds to the zero Lyapunov exponent, while negative exponents correspond to the amplitude variables. Starting from this point, we want to define phase of a continuous-time dynamical system with chaotic behaviour as a variable that corresponds to its zero Lyapunov exponent. We consider three approaches to determination of phase.

A) Sometimes we can find such a projection of the attractor of the system on some plane (x, y) that the plot reminds of the smeared limit cycle, *i.e.* the trajectory rotates around the origin (or any other point that can be taken as the origin). It means that we can choose the Poincaré section in a proper way. With the help of the Poincaré map we can thus define a phase, attributing to each rotation the 2π phase increase:

$$\phi_{\mathrm{M}} = 2\pi \frac{t - t_n}{t_{n+1} - t_n} + 2\pi n, \qquad t_n \le t < t_{n+1} \,, \tag{4}$$

where t_n is the time of the n-th crossing of the secant surface. Note that for periodic oscillations this definition gives the correct phase satisfying eq. (3). Defined in this way, the phase is a piecewise-linear function of time. It is clear that shifts of this phase do not grow or decay in time, so it corresponds to the direction with the zero Lyapunov exponent. However, this phase crucially depends on the choice of the Poincaré map. Here we face the same problem as in the choice of the appropriate variable used for definition of Hilbert transform phase ϕ_H (see below).

B) If the above-mentioned projection is found, we can also introduce the phase as the angle between the projection of the phase point on the plane and a given direction on the plane (see also [11, 20]):

$$\phi_P = \arctan(y/x)\,. \tag{5}$$

Note that although the two phases ϕ_M and ϕ_P do not coincide microscopically, *i.e.* on a time scale less than the average period of oscillation, they have equal average growth rates. In other words, the mean frequency defined as the average of $d\phi_P/dt$ over large period of time coincides with a straightforward definition of the mean frequency via the average number of crossings of a Poincaré surface per unit time.

C) One can often find an "oscillatory" observable that provides the Hilbert phase ϕ_H in agreement with our intuition. For example, the z-variable is a natural choice for the Lorenz system.

The comparison of ϕ_M, ϕ_P and ϕ_H for several examples is given in [13]: it was shown that at least for topologically simple, *e.g.*, Rössler and Lorenz attractors, these approaches produce nearly coinciding results.

3. – Phase synchronization of chaotic self-sustained oscillators

In this section we demonstrate the effect of phase synchronization of *chaotic* oscillators. Using the above-described definitions of phase we show that the interaction of non-identical autonomous chaotic oscillators can lead to a perfect locking of their phases, whereas their amplitudes remain chaotic and non-correlated.

To study phase synchronization of coupled chaotic oscillators, we calculate their phases and check whether the locking condition $|n\phi_1 - m\phi_2| <$ const is satisfied; here we restrict ourselves to the case $m = n = 1$.

First we demonstrate an example, where the phase satisfying eq. (3) can be introduced rigorously, and the possibility of phase synchronization is obvious. We consider the oscillator described by

$$\begin{cases} \dot{x} = \dfrac{e}{2}x - \omega y - z\dfrac{x^2}{x^2+y^2}\,, \\ \dot{y} = \dfrac{e}{2}y + \omega x - z\dfrac{xy}{x^2+y^2}\,, \\ \dot{z} = f + z(x - c)\,, \end{cases} \tag{6}$$

where e, f, c, and ω are parameters. This system can be considered as a modification of the well-known Rössler system (see below) with the same parameters, and its attractor

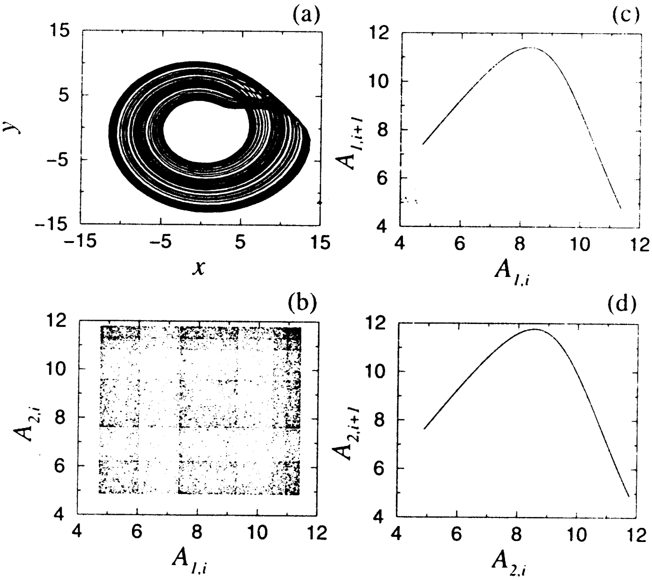

Fig. 1. - Projection of the attractor of the system (6) for the parameter values $f = 0.4$, $e = 0.15$, $c = 8.5$, and $\omega = 1$ (panel (a)). The amplitudes $A_{1,i}$ and $A_{2,i}$ of two synchronized systems (8) for $\omega_0 = 1$, $\Delta = 0.02$, $\varepsilon = 0.05$ are independent (panel (b)), although each amplitude remains chaotic, as can be seen from the next amplitudes plots (c) and (d).

(fig. 1 (a)) is similar to the Rössler attractor. By substitution $x = A\cos\phi$ and $y = A\sin\phi$, eqs. (6) can be rewritten in the form

$$\left\{\begin{aligned} \dot{A} &= \frac{e}{2}A - z\cos\phi\,, \\ \dot{\phi} &= \omega\ , \\ \dot{z} &= f + z(A\cos\phi - c)\ . \end{aligned}\right. \tag{7}$$

The second equation in (7) coincides with eq. (3) governing the dynamics of the phase of a periodic oscillator; the phase of the chaotic system (6) introduced in this way obviously corresponds to the zero Lyapunov exponent. Suppose two oscillators of this kind are coupled via the phase variables

$$\left\{\begin{aligned} \dot{A}_{1,2} &= \frac{e}{2}A_{1,2} - z_{1,2}\cos\phi_{1,2}\,, \\ \dot{\phi}_{1,2} &= \omega_{1,2} + \varepsilon\sin(\phi_{2,1} - \phi_{1,2})\,, \\ \dot{z}_{1,2} &= f + z_{1,2}(A_{1,2}\cos\phi_{1,2} - c)\,, \end{aligned}\right. \tag{8}$$

where $\omega_{1,2} = \omega_0 \pm \Delta$. Then, obviously, the phases $\phi_{1,2}$ are locked if the coupling strength $\varepsilon \geq \Delta$. If the frequency mismatch is small, $\Delta \to 0$, the locking takes place for vanishing coupling. Hence, similar to the synchronization of periodic oscillators and contrary to

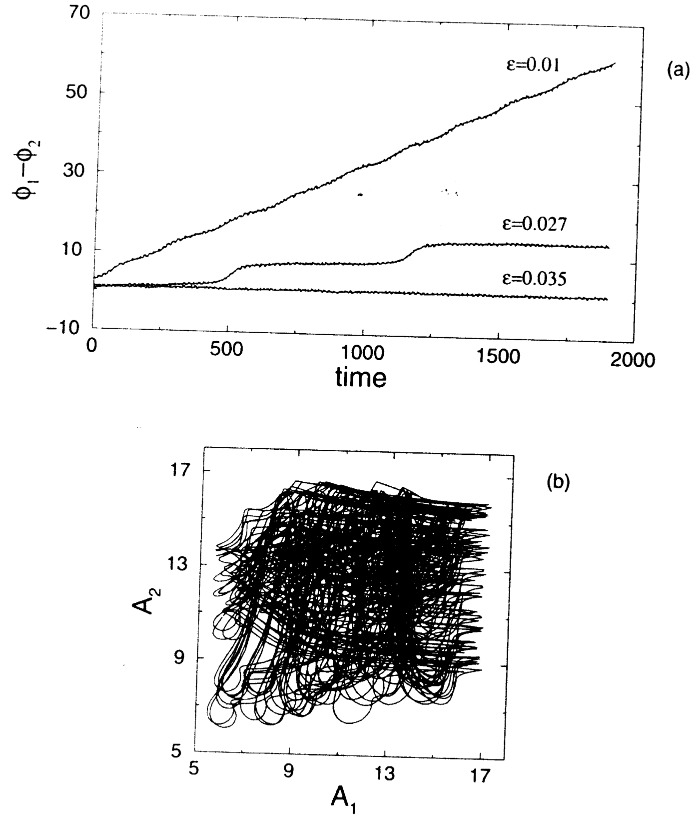

Fig. 2. – Phase difference of two coupled Rössler systems (eq. (9)) *vs.* time for non-synchronous ($\varepsilon = 0.01$), nearly synchronous ($\varepsilon = 0.027$) and synchronous ($\varepsilon = 0.035$) states, (a). In the last case the amplitudes $A_{1,2}$ remain chaotic, (b), their cross-correlation is less than 0.2. The frequency mismatch is $\Delta = 0.015$.

other types of synchronization of chaotic systems, phase synchronization appears *without threshold.*

To demonstrate the chaotic character and independence of amplitudes $A_{1,2}$ in the synchronous state, we calculate them at time moments $2\pi/\omega_0 \cdot n$, *i.e.* for the constant values of $\phi_{1,2}$, construct the next amplitude plots (partial Poincaré maps) and plot one amplitude *vs.* the other one (fig. 1).

As the second example of phase synchronization, we consider two coupled Rössler systems [21]

$$\left\{\begin{aligned} \dot{x}_{1,2} &= -\omega_{1,2}y_{1,2} - z_{1,2} + \varepsilon(x_{2,1} - x_{1,2})\,, \\ \dot{y}_{1,2} &= \omega_{1,2}x_{1,2} + ey_{1,2}\,, \\ \dot{z}_{1,2} &= f + z_{1,2}(x_{1,2} - c)\,. \end{aligned}\right. \tag{9}$$

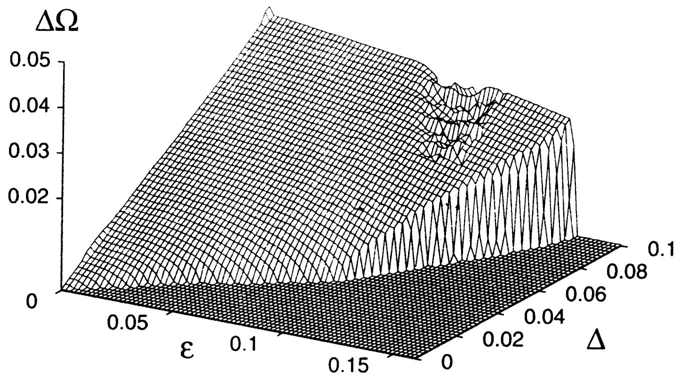

Fig. 3. – The mean frequency difference Δ for the coupled Rössler systems (9), calculated with the method of partial Poincaré maps, as a function of the coupling ε and the frequency mismatch Δ. For ε large enough the frequency difference $\Delta\Omega$ is nearly zero; this region of synchronization is completely analogous to the phase-locking domain (the Arnold tongue) for coupled periodic oscillators. For small ε there is no synchronization and the phase difference grows with the finite rate $\Delta\Omega$.

Here we introduce the parameters $\omega_{1,2} = 1 \pm \Delta$ and ε which govern the frequency mismatch and the strength of coupling, respectively([1]); $f = 0.2$, $c = 10$, $e = 0.15$. Here we are not able to introduce the phases rigorously, and we calculate them as well as the amplitudes by means of the Hilbert transform from the variables $x_{1,2}$. As the coupling is increased for a fixed mismatch Δ, we observe a transition from a regime, where the phases rotate with different velocities $\phi_1 - \phi_2 \sim \Delta\Omega \cdot t$, to a synchronous state, where the phase difference does not grow with time $|\phi_1 - \phi_2| <$ const; $\Delta\Omega = 0$. This transition is illustrated in fig. 2 (a).

For the Rössler attractor, because of its simple form, it is very convenient to calculate the phases basing on the Poincaré map construction. When we consider coupled chaotic systems, we still can construct partial Poincaré maps, *e.g.*, taking successive maxima of the variables $x_{1,2}$ in the coupled Rössler systems. Partial frequencies are then simply defined as an average number of crossings of the secant surfaces per unit time. According to this approach, the synchronization in coupled Rössler systems simply means that the average numbers of oscillations (number of maxima) per unit time in both systems coincide. The region of synchronization in the plane of parameters "coupling-frequency mismatch", obtained using these partial Poincaré maps, is presented in fig. 3. Note that it seems to have no threshold. This is a particular feature of the Rössler system, where the motion is highly coherent (in the power spectrum a very sharp peak is observed [22]).

([1]) One can see that $\omega_{1,2}$ are indeed frequencies of the Rössler system if we rewrite it as

$$\ddot{y} - a\dot{y} + \omega^2 y = -\omega z\,, \qquad \dot{z} + fz = b + z(\dot{y} - ay)z/\omega\,.$$

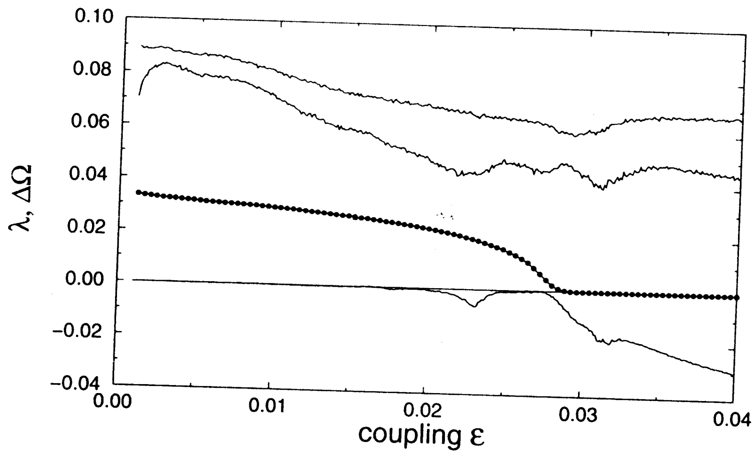

Fig. 4. – The four largest Lyapunov exponents, one of which is always zero (lines) and $\Delta\Omega$ (circles) *vs.* coupling ε for system (9) with $\Delta\omega = 0.015$.

On the other side, it is possible to synchronize systems with frequency mismatch of more than 20% (see fig. 3).

It is remarkable how the phase synchronization manifests itself in the Lyapunov spectrum (fig. 4). In the absence of coupling, each oscillator has one positive, one negative, and one vanishing Lyapunov exponents. As the coupling is increased, the positive and the negative exponents remain, whereas one of the zero exponents becomes negative. This behaviour can be explained as follows: without coupling, the vanishing exponents correspond to the translation along the trajectory, *i.e.* to the shift of the phase of the oscillator. The coupling produces an "attraction" of the phases such that the phase difference $\phi_1 - \phi_2$ decreases. Thus one of the vanishing exponents becomes negative. For large coupling the attraction is so strong that the phases remain locked.

4. – Looking for synchronization phenomena in real data

The real word is often (if not always) non-stationary. Parameters of interacting subsystems and/or of coupling may vary with time. Nevertheless, as the stationarity of the time series is not required for the Hilbert transform, we can calculate the phase difference and find epochs of synchronous and non-synchronous behaviour.

We illustrate this by the example of two coupled Rössler systems (9) with slowly varying coupling strength $\varepsilon = 0.03 + 0.02\sin(0.01t)$; parameters $\omega_1 = 0.89$ and $\omega_2 = 0.85$. Due to modulation of the coupling, oscillators synchronize and desynchronize repeatedly. Then, calculating the relative phase between $x_1(t)$ and $x_2(t)$ (fig. 5), we can easily distinguish time intervals, where the phase difference is constant, *i.e.* phases are locked. Hence, from these bivariate data we can conclude that within these intervals there is a resonant interaction between the systems, and they are synchronized. Similar results can be obtained if two completely different systems, namely periodic van der Pol oscillator and chaotic Rössler systems, are coupled [23].

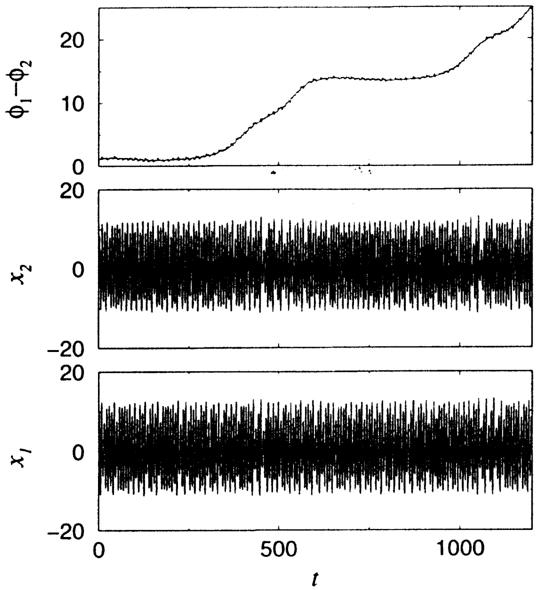

Fig. 5. – Phase difference between two coupled Rössler oscillators. The coupling coefficient changes slowly with time. The periods of synchronous motion can be clearly seen.

As the second example we present the result of experiments on posture control in neurological patients [24]. During these tests a patient is asked to stay quietly on a special rigid force plate with four tensoelectric transducers. The output of the setup provides current coordinates (x, y) of the centre of pressure under the feet of the standing subject. These bivariate data are called stabilograms; they are known to contain rich information on the state of the central nervous system. In the following we denote the deviation of the centre of pressure in anterior-posterior and lateral direction as x and y, respectively. Every subject was asked to perform three tests of quiet standing with a) eyes opened and stationary visual surrounding (EO); b) eyes closed (EC); c) eyes opened and additional video-feedback (AF). In order to eliminate low-frequency trends, the moving average computed over the n-point window was subtracted from the original data. The window length n has been chosen by trial to be equal or slightly larger than the characteristic oscillation period. Its variation up to two times does not practically effect the results.

Here we present in detail results of the analysis of one trial (female subject, 39 years old, functional ataxia). We can see that in the EO and EC tests the stabilograms are clearly oscillatory (fig. 6). The difference between these two records is that with eyes opened the oscillations in two directions are not synchronous during approximately the first 110 s, and are phase locked during the last 50 s. In the EC test, the phases of oscillations are perfectly entrained during all the time. The behaviour is essentially different in the AF test; here no phase locking is observed.

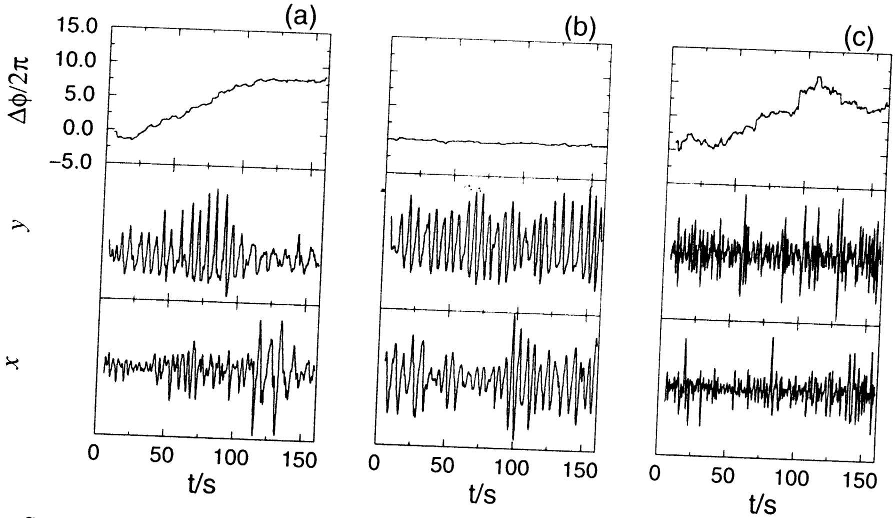

Fig. 6. – Stabilograms of a neurological patient after trend elimination for (a) EO, (b) EC, and (c) AF tests. The upper panels show the relative phase between two signals x and y. During the last 50 s of the first test and the whole second test the phases are perfectly locked. No phase entrainment is observed in the AF test.

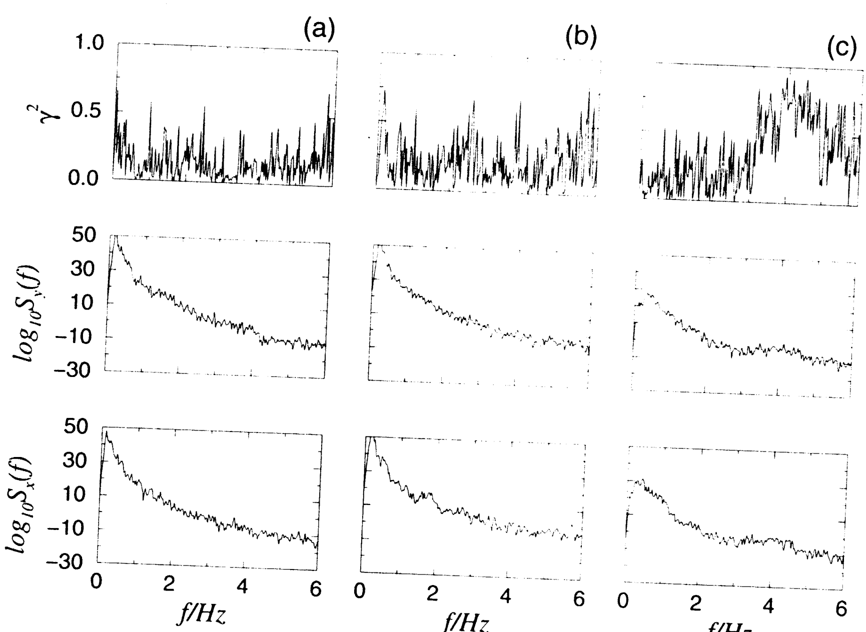

Fig. 7. – Auto-spectra and coherence functions for the stabilograms shown in fig. 6. Although the time series in the EC tests are perfectly phase locked, the spectral analysis shows no significant coherence.

It is noteworthy that conventional methods of time series analysis are not efficient in this case. So, to quantify the linear correlations between x and y in the frequency domain we have calculated cross-spectrum and have obtained from it the coherence function (fig. 7). We see that, although the low-frequency peaks in the spectra are clearly seen, the coherence is not very high ($\gamma^2 \approx 0.5$ for the EO test and $\gamma^2 \approx 0.7$ for the EC test). The characteristic of non-linear relationship, the generalized mutual information function [15, 25], also does not reveal significant dependence, even in the EC test, where the phases are completely locked.

Applications of this method to the analysis of the cardio-respiratory system of a piglet allowed us to find presence of epochs of phase locking between respiration and heart rate variability for certain physiological states; these results are presented in [26].

5. – Conclusions

We have demonstrated the possibility of phase synchronization of chaotic self-sustained oscillators. In this regime the phases are synchronized, while the amplitudes vary chaotically and are practically uncorrelated. The effect of phase synchronization is also possible when the natural frequencies are in a rational relation (this is relevant for an important physiological problem of interaction of the cardiac and the respiratory systems).

We emphasize that the phase synchronization is observed already for extremely weak couplings, and in some cases can have no threshold, contrary to other types of synchronization. This phenomenon is a direct generalization of synchronization of periodic self-sustained oscillators. As the latter, it may find practical applications, in particular when a coherent summation of outputs of slightly different generators operating in a chaotic regime is necessary. For this purpose, it is sufficient to synchronize phases, while amplitudes can remain uncorrelated. We expect this to be relevant for an important problem of outputs summation in arrays of semiconductor lasers [27]. For the description of such arrays, as well as of a number of other physical and biological phenomena, one often uses a model of globally coupled oscillators. Here mutual phase synchronization of individual chaotic states manifests itself as an appearance of a macroscopic mean field [11].

We have described a consistent method of calculation of the phase difference between two time series. We have shown that this method can be effectively used to reveal time-varying weak interaction between self-oscillating systems, which can be either chaotic or periodic.

Let us stress that if the phase difference between components of bivariate data is bounded, it does not necessarily mean that the signals are generated by two mutually synchronized oscillatory systems. For example, these signals can be the input and output of some phase-shifting (non-linear) filter, or originate from two oscillators entrained by a third one. Nevertheless, the technique can be formally applied; both the assumption on the underlying model and the interpretation of the result depend on the particular problem. This is similar to the usage of the coherence function and phase of the cross-spectra: although the model underlying cross-spectrum calculation is a one-input–one-output linear system, the technique is often applied to arbitrary bivariate data.

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