Comment on "Chaos, Noise, and Synchronization"

In a recent Letter Maritan and Banavar [1] have reported the effect of synchronization of two noninteracting chaotic systems subject to the same noise. They have considered two systems, the noisy logistic map and the noisy Lorenz equations, and have shown that for some values of the noise intensity the distance between the states of two identically driven systems becomes zero.

Ensembles of dynamical systems governed by the same external noise have been considered in Refs. [2,3]. In these papers a criterion for synchronization of these ensembles has been formulated: Synchronization occurs if the largest Lyapunov exponent is negative; if the exponent is positive, the systems are desynchronized. This criterion is valid for any size of the ensemble, in particular, for ensembles consisting of two components, as in [1]. Note also that the largest Lyapunov exponent can be calculated from a single system, although the nontrivial effect of synchronization can be observed only for ensembles.

Now, we apply the results of Refs. [2,3] to the simplest system discussed by Maritan and Banavar, namely, to the noisy logistic map (see [1] for description of the system, we will follow the notations of [1]). We have calculated the largest Lyapunov exponent for the same parameters as in [1] and found that it is positive in the whole range of noise intensity. Thus, the findings of [1] disagree with the predictions of [2,3]. To resolve this we have simulated the dynamics using different precisions of calculation. In the case of double precision (64 bits), in accordance with [1], we have observed synchronization for the noise amplitude $W = 0.6$ (within 100 realizations in 99 cases the states of the two systems become identical after 5×10^5 iterations). However, in the case of quadrupole precision (128 bits) no signs of synchronization have been observed (not any identical state within 1000 realization after 5×10^5 iterations), but the averaged distance saturates at $\overline{d^2} \approx 0.08$. Thus, the synchronization appears only if the precision of the calculations is not very high.

In fact, Maritan and Banavar have discussed the effect of finite precision. They claim that with finite precision ε (i.e., when the distance between the trajectories d becomes smaller than ε, it is set to zero and remains zero thereafter) a critical value of noise intensity exists, which separates synchronous and nonsynchronous regimes. This is not the case if one considers sufficiently large time intervals. Indeed, in systems with noise and finite phase space there is always a nonzero probability that the distance between two trajectories is smaller than ε (one can estimate this probability as $\sim\varepsilon^D$, where D is the topological dimension of the phase space). Thus, after sufficiently large time, the two systems will be synchronized. This "finite-precision-synchronization" will be, however, destroyed by small nonidentity of the systems (we remind that a small discrepancy grows due to positiveness of the largest Lyapunov exponent), and therefore will be not observed in real experiments. The synchronization described in Refs. [2,3] is, in contrast, stable to perturbations (a small discrepancy decreases in time because the largest Lyapunov exponent is negative).

In conclusion, the phenomenon of synchronization observed in [1] is a numerical effect of the insufficient precision of calculations. In real systems the criterion suggested in [2,3] should be applied.

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