CHAOS IN A SOLID-STATE LASER WITH PERIODICALLY MODULATED LOSSES

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It is shown theoretically that an experimentally observable chaotic behavior may arise in a single-mode solid-state laser when its losses are periodically modulated with a frequency less than half the frequency of the relaxation oscillations.

Quite a number of laser models with chaotic behavior have appeared recently. The most famous example is the Lorenz equations [1] which coincide with the equations for a single-mode laser [2,3]. In a similar system, chaos was numerically obtained by Grasyk and Oraevsky [4]. Multimode chaotic oscillations were reported for lasers with a saturable absorber [5] and without it [6]. In a number of papers [7–9,11] nonautonomous laser dynamics was investigated. In refs. [8,9] the additional time dependence in a nonautonomous laser system was assumed to be caused by low-frequency periodic variations of the external field or the pump. It should be emphasized that in refs. [8,9] an adiabatic approximation was used (the inversion and polarization of an active medium follow the field amplitude). Thus, the physical mechanism responsible for chaos was nonlinear interaction of two frequencies: detuning of the external field frequency from that of an autonomous laser and the low frequency of the pump (or the external field amplitude) variations.

However, an adiabatic approximation does not hold for a solid-state laser. In this situation there is a characteristic frequency (that is the frequency of relaxation oscillations [10]). Thus, in such a system chaos can be expected to emerge without an external field, provided the laser parameters are periodically modulated.

In the present letter we show chaos to exist in a single-mode solid-state laser with periodically varied losses. A similar system was studied in ref. [11] but the numerical results presented are too scanty to draw a reliable conclusion on the existence of chaos.

The governing equations are [12]:

$$dE/dt = -\kappa(1 + \beta \cos \omega t)E + i\rho_1 P,$$

$$dP/dt = -P/T_2 - i\rho_2 DE,$$

$$dD/dt = -(D - D_0)/T_1 - 2i\rho_3(PE^* - EP^*), \qquad (1)$$

where E, P, D are the field and polarization amplitudes and the inversion; $\kappa(1 + \beta \cos \omega t)$ describes the periodically modulated losses. For solid-state lasers one has $T_2 \ll T_1$, so the polarization can be excluded by setting $dP/dt = 0$. Then for the dimensionless intensity of radiation $I = 4\rho_2 \rho_3 T_2 EE^*$, the inversion $n = \rho_1 \rho_2 T_2 D/\kappa T_1$ and the time $\tau = t/T_1$ we obtain:

$$\mathring{I} = GI(n - 1 - \beta \cos \Omega\tau), \quad \mathring{n} = \alpha - n(I + 1), \qquad (2)$$

where

$$G = 2\kappa T_1, \quad \alpha = \rho_1 \rho_2 T_2 D_0/\kappa T_1, \quad \Omega = \omega T_1.$$

One can consider system (2) as a nonlinear oscillator driven by a periodic external force with a frequency Ω. In similar systems, chaos was investigated in refs. [13,14].

If $\beta = 0$, system (2) has a globally attracting fixed point $n^0 = 1$, $I^0 = \alpha - 1$. All nearby solutions damp oscillating with a relaxation frequency $\Omega = [G(\alpha - 1)]^{1/2}$. It is evident that nontrivial effects may appear only if the external frequency Ω is not far from Ω_0. In computer experiments we used $G = 10^3$, $\alpha = 2$, $\Omega = 0.4\ \Omega_0$ and β was varied. For small values of β, periodic oscillations were obtained. As β increased, more and more complicated patterns ap-

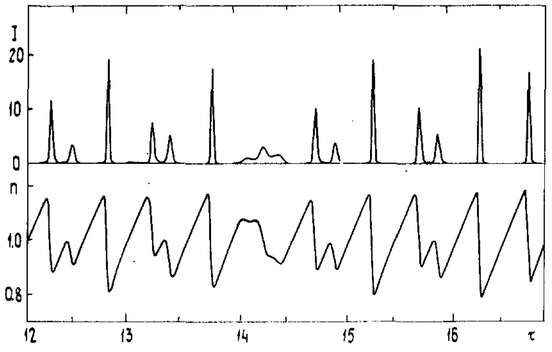

Fig. 1. Time evolution of the field intensity I and the inversion n in the chaotic state ($\beta = 0.075$).

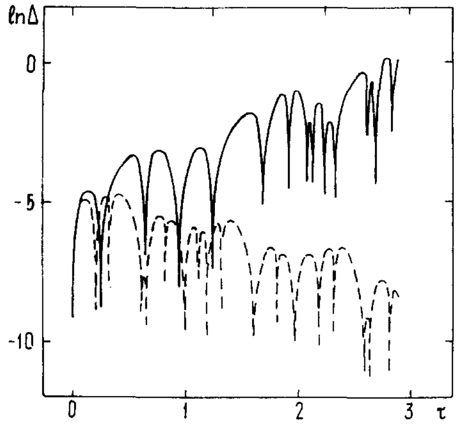

Fig. 2. Time evolution of the distance Δ between initially close orbits: solid line $\beta = 0.075$ (chaotic state); dashed line $\beta = 0.05$ (regular state).

peared (we will not discuss here a detailed structure of bifurcations) and at $\beta = 0.075$ chaotic, erratic oscillations were obtained (fig. 1). It is clear from fig. 1 that radiation has a form of a chaotic sequence of pulses, i.e. the "oscillator" (2) operates in a highly nonlinear regime.

For more convincing evidence of chaos, a criterion of exponential growth of small disturbances in initial conditions is often used. In our situation it is convenient to use for the "distance" between the points (n_1, I_1) and (n_2, I_2) the expression

$$\Delta = [(n_1 - n_2)^2 + (\ln I_1 - \ln I_2)^2]^{1/2}$$

due to the existence of deep minima of I.

Time evolution of Δ in the chaotic and regular states is presented in fig. 2. Exponential growth of Δ confirms reliably that system (2) has a strange attractor. At large β the oscillations become periodic again. Note that the regions both of β and Ω at which stochasticity is observed are rather small.

In conclusion we would like to emphasize that we have considered a realistic model of a solid-state laser. For example, in the case of a Nd^{3+}: YAG laser $T_1 \approx 2 \times 10^{-4}$ s, $T_2 \approx 10^{-12}$ s, $\kappa = 10^6 - 10^7$ s^{-1}, thus, $G \approx 10^3$ and the pump level $\alpha = 2$ can be easily achieved experimentally. One can expect that the proposed mechanism of chaos (i.e. nonlinear interaction of relaxation frequency and frequency of external modulation of losses) holds also for multimode laser systems, for example, ring lasers.

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