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STOCHASTIC OSCILLATIONS IN DISSIPATIVE SYSTEMS

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The recent discovery of simple deterministic systems, the behaviour of which is intrinsically stochastic, has led to a new approach to the problem of turbulence. This paper is devoted to the discussion of some physical mechanisms for the appearance of chaos in simple dissipative systems. The authors show that stochastic behaviour is the result of the self-disorganization of such systems. Examples from electronics, chemistry, theory of nonlinear oscillations and waves are given as illustrations to the theoretical findings.

INTRODUCTION
CONSERVATIVE AND DISSIPATIVE MECHANISMS FOR CHAOS

Many physical systems exhibit chaotic (irregular) behaviour. The most well-known example can be found in statistical mechanics, in which statistical properties of big sets of particles, governed by laws of Hamiltonian mechanics, are investigated (it should be noted that the present paper deals only with classical, but not quantum systems). Another example is turbulence in a fluid, which is quite troublesome for engineers.

The main difference of statistical mechanisms from theory of turbulence lies in the fact that the latter deals with nonequilibrium systems, the chaotic behaviour of which is characterized not only by the nature of interaction of separate subsystems but by the character of sources and sinks of energy as well. The common feature of these disciplines is that both study extremely complex objects consisting of large numbers of subsystems. That is why the statistical description of the problem discussed appeared to be not only natural but the only possible as well.

Recent fifteen years have considerably changed the understanding of the nature of stochastic behaviour in physical systems. Firstly, simple regular behaviour was discovered in many complex systems that made investigators undertake special efforts for determining mechanisms, generating chaos. Secondly, examples of simple dynamical systems with stochastic behaviour were found. The investigation of such systems enabled to clarify the relation between determinism and randomness; this relation has been a very obscure problem in classical physics for quite a long period of time. The above-said is illustrated by the following table.

		Regular Behaviour	Irregular Behaviour
Simple Systems	Conservative	Nonlinear periodic oscillations (a pendulum)	Ergodic motions (Henon-Heiles Hamiltonian, Billiards, etc.)
	Dissipative	Periodic self-sustained oscillations (Van-der-Pol generator and other systems with a limit cycle)	Stochastic self-sustained oscillations (the Lorenz model and other systems with a strange attractor)
Complex systems	Conservative	Integrable and near-integrable systems (KdV and Sine-Gordon equations, KAM-theory, etc.)	Ideal gas (Sinai's system of hard spheres in a box)
	Dissipative	Self-organized ordered dissipative structures (Benard cells and other examples from Synergetics)	Developed turbulence in fluid and plasma

Conservative systems are separated from dissipative ones everywhere in the table. The reason lies in the fact that in these two cases physical mechanisms for both regular and irregular behaviour are completely different. Stochasticity in a dissipative system should be regarded as a result of self-desorganization, as a limiting regime, independent of the initial conditions and hardly sensitive to external perturbations. Different conservative mechanisms for chaos (overlapping of resonances and others [1,2]) differ from one another only by the nature of nonlinear interaction of modes (degrees of freedom) and by the correlation of frequences. But in a nonequilibrium system the nature of stochasticity is determined aditionally by the properties of sources and sinks of energy. Naturally, the source and sink might be introduced in the way that they do not affect purely conservative stochasticity [3], but this is supposed to be a very degenerate case.

We shall try in this paper to describe a number of essentially dissipative mechanisms for stochasticity. Particular attention will be given to one of them, and more strictly, to its realization in an electronic noise generator and in a chemical Belousov-Zhabotinsky reaction. Then the chaotic states of systems of coupled growing and damping nonlinear oscillators will be discussed. Other mechanisms, working, for example, in the Lorenz system, are dealt with in a very brief manner.

SWITCHING MECHANISM FOR CHAOS

Simple noise generator. Historically the theory of periodic self-oscillations developed in a close interrelation with electronics. It was discovered that self-sustained periodic oscillations in an electronic circuit correspond to a limit cycle in the phase space of the equations describing the circuit. The limit cycle is a simple attractor, but recently strange attractors appeared, i.e., attracting sets in phase space, which stochastic self-sustained oscillations correspond to [5]. Like in the case of the limit cycle, the simplest realization of the strange attractor is an electronic circuit, which, in this case, is naturally regarded as a noise generator. For this reason the description of the switching mechanism for chaos will be illustrated by a simple noise generator, shown in Fig.1. As it is shown further, this mechanism also works in systems of different nature, for example, in a chemical Belousov-Zhabotinsky reaction.

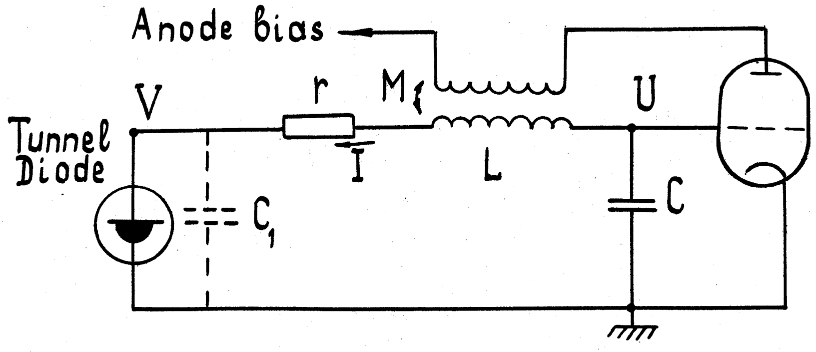

Figure 1
Electronic circuit of a simple noise generator

This mechanism is very similar to the "classical" Van-der Pol generator. The only difference is a tunnel diode, inserted in series with an inductor. But this "little" difference drastically changes the generator's behaviour: the vacuum tube (or some other similar active element, for example, a transistor) works in a purely linear regime and serves as a source of energy, and the only non-linearity, providing limitation of oscillations and stochasticity of the output, is the non-linearity in the tunnel diode voltage-current characteristic (Fig. 2).

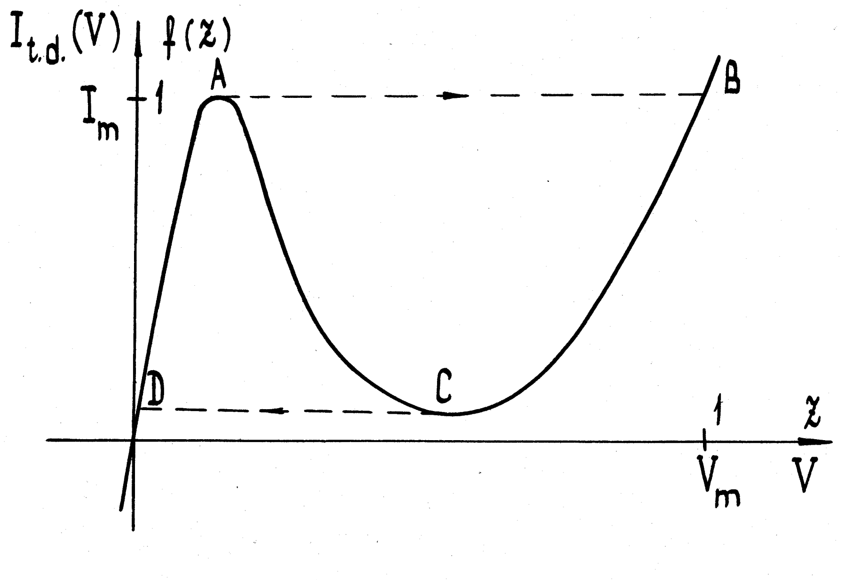

Figure 2
Tunnel diode voltage-current characteristic

Taking into consideration, as it is usually done, the small capacitance of the tunnel diode C_1 and applying Kirchhoff's laws to the circuit of Fig. 1, we obtain

$$LC\frac{dI}{dt} = (MS - rC)I + C(U - V) \quad (1a)$$

$$C\frac{dU}{dt} = -I \quad (1b)$$

$$C_1\frac{dV}{dt} = I - I_{t.d.}(V), \quad (1c)$$

where S is the tube transconductance.

It is rather easy to describe processes in such a generator qualitatively. While the current I and the voltage U are small, the tunnel diode does not affect substantially oscillations in the circuit ($V \approx 0$ and the equations (1a),(1b) are not connected with (1c)), and they increase due to the tube energy. When the current I exceeds the threshold I_m, the diode almost instantly switches along the dashed line AB, shown in Fig.2 (the time of this switch is proportional to the small capacitance C_1), and the diode voltage becomes V_m. Then the current I decreases, and the diode returns to the state with $V \approx 0$ along the dashed line CD. In the result of these two switches the diode consumes part of the circuit energy; after that the oscillations begin to increase again, the current I exceeds the value I_m and so on. Thus, the output signal U(t) appears to be a sequence of impulses of exponentially increasing oscillations (see Fig.5 below), the voltage V(t) is a sequence of narrow peaks with the amplitude $V \approx V_m$; between the peaks V is small.

The above qualitative description does not permit to find out whether the oscillations are periodic or stochastic. This can be determined with the help of mathematical analysis of the equations(1).

Mathematical treatment. Introducing dimensionless variables $x = I/I_m$, $y = UC^{1/2}/(I_mL^{1/2})$, $z = V/V_m$, we shall put down the equation (1) in the following form

$$\dot{x} = 2\cdot hx + y - gz \quad (2a)$$

$$\dot{y} = -x \quad (2b)$$

$$\varepsilon\dot{z} = x - f(z), \quad (2c)$$

where $h = 0{,}5\cdot(MS - rC)(LC)^{-1/2}$ is the growing increment; $g = V_mC^{1/2}/(I_mL^{1/2})$ is the characteristic parameter; $\varepsilon = gC_1/C \ll 1$; $f(z) = I_{t.d.}(V_mZ)/I_m$ is the dimensionless charecteristic of the diode (see Fig.2); and the dot denotes the derivative over the dimensionless time $\tau = t(LC)^{-1/2}$.

The mathematical treatment of the equations (2) is simplified by the fact that there is a small parameter ε in front of the derivative in (2c). Therefore all motions in the phase space can be approximately divided into fast and slow ones. The slow motions are restricted to the slow manifold $x = f(z)$, and only to its stable branches, where $f'(z) > 0$. The fast motions are the straight lines x = const, y = const, and z goes over from one stable branch of the slow manifold to another [6]. The slow motions correspond to the oscillations in the circuit, and the fast ones - to the switches of the diode.

So, the phase space of the system (2) degenerates in a pair of overlapping slow surfaces, connected by fast motions. (It should be noted that other strange attractors in 3-dimensional systems, for example, the Lorenz attractor, are also almost two-dimensional [7], but the corresponding surfaces have a complicated form and can be determined only numerically).

Now we may try to solve the equations (2a) and (2b) on slow surfaces and then connect these solutions with fast motions. But this will not lead us much closer to the understanding of stochasticity, for where there is a solution, there is sheer determinism, but not stochasticity. Therefore we have to resort to a reductive description by cutting out deterministic non-stochastic parts of the trajectory. This "cutting out" is usually effected with the help of the Poincaré map. To construct it, we take a line on the slow surface, for example $\Sigma\{Y = y | x = 0, y > 0\}$; emerge a trajectory from every point and mark a point on Σ, in which the trajectory returns. Thus, we shall get the Poincaré map $\bar{Y} = F(Y)$. The obtained dynamical system with discrete time preserves all the properties of the equations (2) and, consequently, all the properties of the electronic noise generator. But its investigation is much simpler than that of the initial equations.

Now we shall go over to constructing the Poincaré map. For every trajectory, leaving Σ, there are two possible ways of behaviour. If Y is sufficiently small, the trajectory makes one revolution around the origin and returns onto Σ - for such trajectories $\bar{Y} = F_1(Y)$. And if Y exceeds some critical value, then the trajectory begins to make a revolution, but without completing it reaches the boarder of the slow surface, goes over to another slow surface, moves on it for some time, jumps back and only after that completes the revolution and returns onto Σ - in this case $\bar{Y} = F_2(Y)$. Both variants are shown in Fig.3.

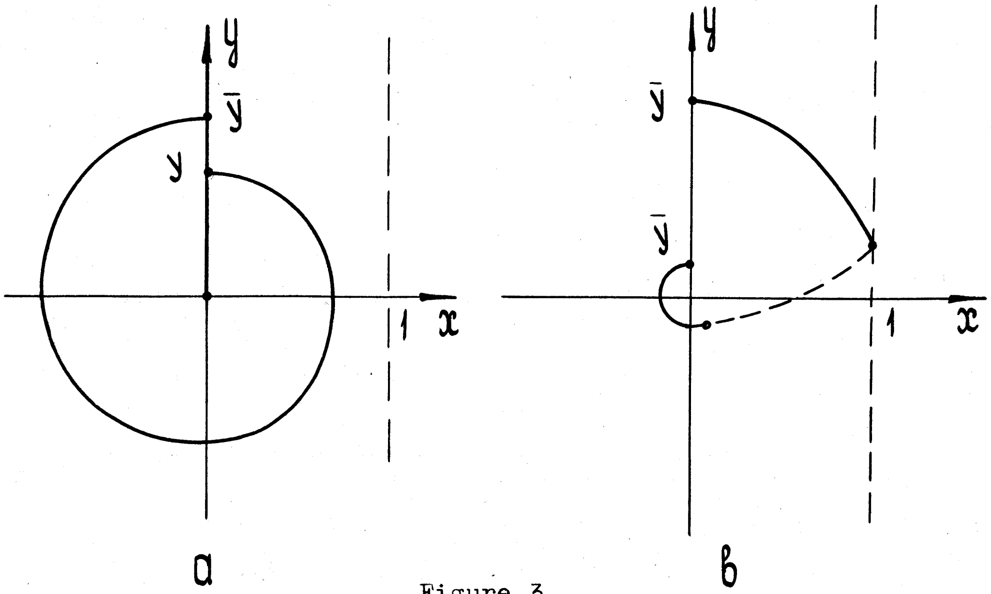

Figure 3
Construction of the Poincaré map:
a) the trajectory lies on one slow surface;
b) the trajectory goes over to another slow surface (broken line)

Since f(z) is a nonlinear function, exact explicit expressions for F_1 and F_2 cannot be obtained. This can be done only by the approximation of f(z) with a piecewise-linear function [8,9]. But even then we shall have rather complicated formulae. Therefore we shall give approximate expressions for F_1 and F_2, which on the whole describe the behaviour of these functions correctly.

In order to get a simple expression for F_1, we shall assume that $z \approx 0$ on the slow surface, so the equations (2a) and (2b) are easily solved:

$$\bar{y} = F_1(y) = \exp(2\pi h)\cdot y \equiv T\cdot y, \qquad (3)$$

where $T > 1$. We shall approximate the function F_2 by the following formula:

$$\bar{y} = F_2(y) = A - \sqrt{y - K}, \qquad (4)$$

where K is the threshold value of y : when $y < K$, the expression(3) is used, when $y > K$, the expression (4) is used. The presence of a square root reflects the fact that the trajectories approach the boarder of the slow surface $x = 1$ almost along the tangent line. The constant A describes the shift of the trajectories while moving on the second slow surface.

Combining (3) and (4) we arrive at the needed Poincaré map, depicted in Fig. 4.

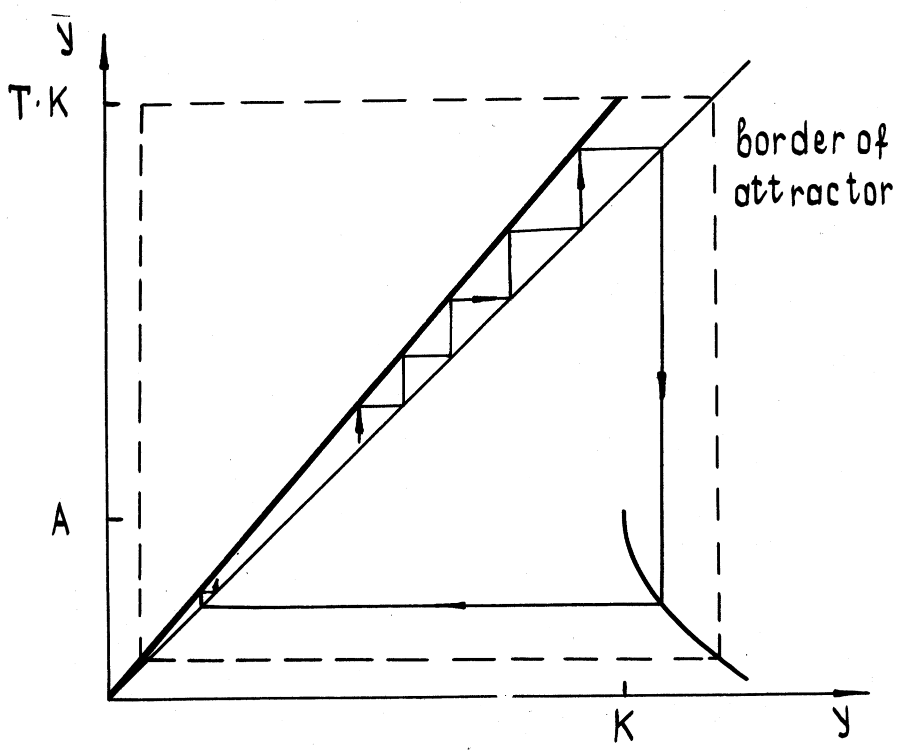

Figure 4
Poincaré map

Let us consider this mapping in detail. First, we shall note that it has the trapping region $A - (TK-K)^{1/2} < y < TK$, from which the trajectories cannot escape. Therefore there is an attractor in the phase space of the system (2). Then, for $0 < T - 1 \leq (4K)^{-1}$ the mapping inside this attractor is expanding, i.e., $|d\bar{y}/dy| > 1$. Hence, there cannot be any stable periodic points in it. Moreover, this mapping is <u>ergodic</u> and <u>mixing</u> [10,11]. Therefore the system (2) should have <u>a strange attractor</u>. Thus, the working regime of the simple generator of Fig. 1 is stochastic, and it can be referred to as a noise generator.

<u>Statistical properties of the output signal</u>. Let us pass over from the mathematical analysis to the real mechanism. Fig.5 depicts the experimentally obtained recording of the output signal.

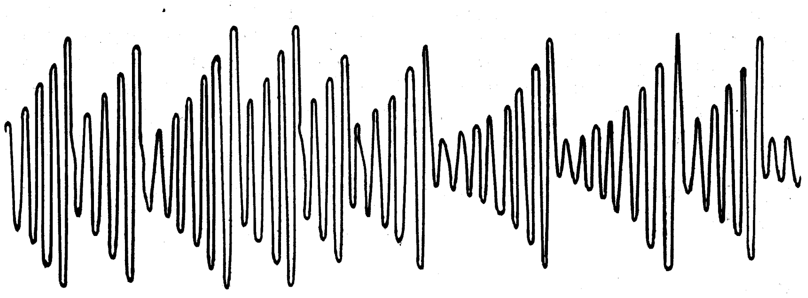

Figure 5
Experimentally obtained recording U(t) of a simple noise generator

This recording is compatible with the above-given qualitative analysis and the Poincaré map. The signal consists of impulses of exponentially increasing oscillations, to which correspond the iterations of the mapping in Fig.4 with $y < K$. When the oscillation amplitude in the impulse exceeds the threshold value ($y > K$), the energy of oscillations decreases sharply, and the transition to a new impulse is executed.

It is evident that the form of each impulse is not random; this is the duration that is random. In the framework of the Poincaré map it is convenient to measure this duration with a discrete variable - the number of maxima in the impulse. In this case the output signal is described by the infinite sequence of random numbers ... n (-1), n(0), n(1), n(2), ..., where n(i) is the number of maxima in the i-th impulse. For the recording of Fig.5 this sequence has the form ...5, 4, 6, 4, 3, 5, 8, 8, 5 ... .

Now let us consider in what way the statistics of this discrete sequence of discrete random quantities can be determined with the help of the constructed mathematical model, i.e., the Poincaré map. The number of maxima in the impulse is equal to the number of iterations of the mapping in Fig.4 with $y < K$. Therefore the statistics is determined by the invariant distribution of probabilities, with respect to which the mapping is ergodic and mixing. This invariant measure is defined by the relation $\mu(\Delta) = \mu(F^{-1}(\Delta))$, where $F^{-1}(\Delta)$ is the full set, for which $F(F^{-1}(\Delta)) = \Delta$. The function F is defined by the relations (3) and (4), and in general the measure μ cannot be expressed analitically. Therefore we shall approximate the function F by a piecewise-linear function, as it is shown in Fig.6. Then the whole attractor is divided into segments

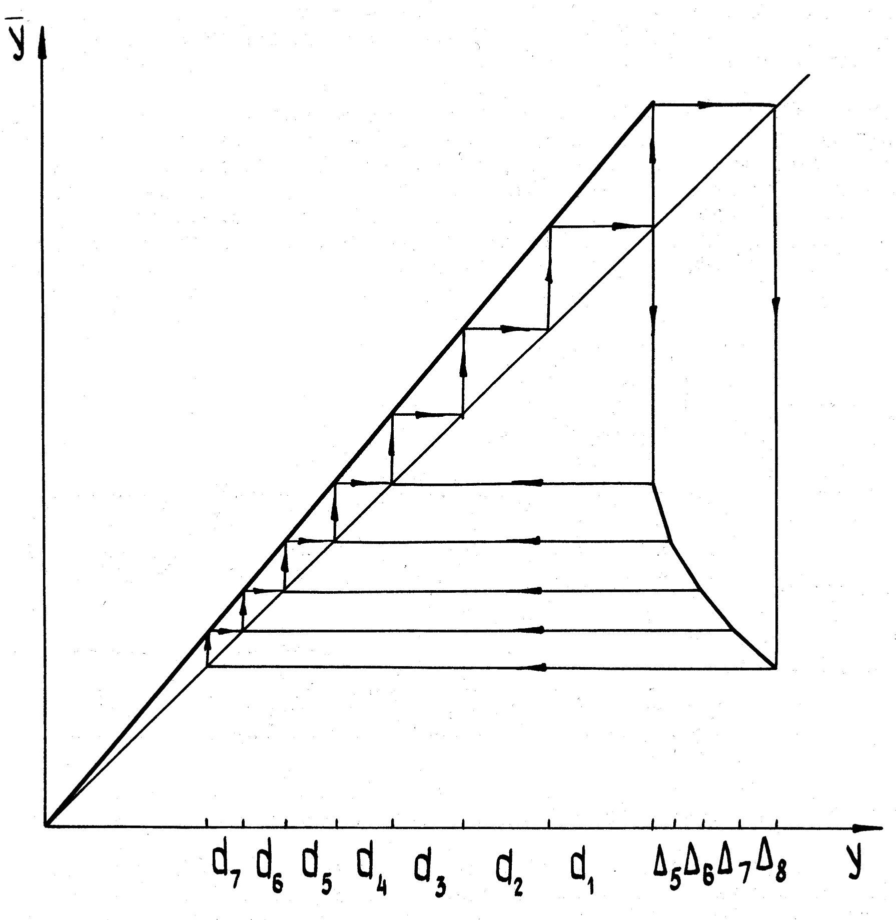

Figure 6
Idealized Poincaré map

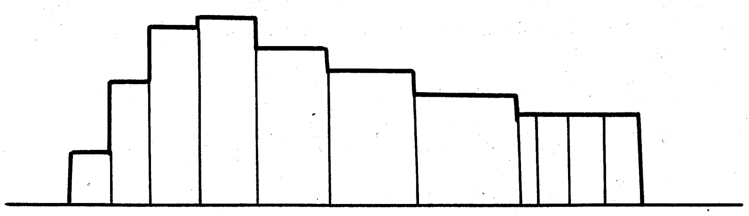

Figure 7
Invariant probability distribution for the map of Fig.6

$d_1 \ldots d_7$ and $\Delta_5 \ldots \Delta_8$, as it is shown in Fig.6(for the sake of clarity we shall deal for some time with concrete numbers and not with general expressions).

Now, if we shall not mark the exact coordinates of the point but the segment, which this point passes, every trajectory will correspond to a sequence of such segments. As it follows from Fig.6, not all the sequences are possible: for example, d_2 always stands before d_1, and after d_1 there follows one of the segments Δ_5 ... Δ_8. One can determine the permissible sequences with the help of the graph of Fig.8.

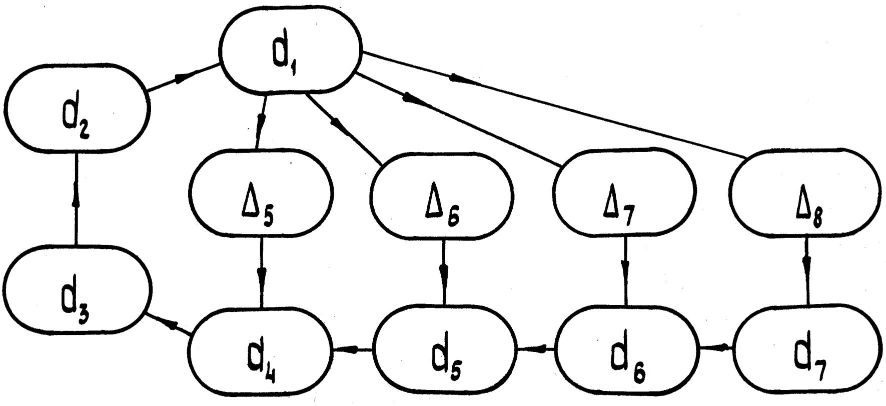

Figure 8
Graph for the mapping of Figure 6

The description of trajectories of a dynamic system with the help of a graph, similar to that of the Fig. 8, is called symbolic, and the corresponding means of investigation are referred to as methods of <u>symbolic dynamics</u> [12]. These methods enable one to describe completely the <u>topological properties</u> of the strange attractor. But we are interested in its <u>metric properties</u>, determining the statistics of the process.

Now, having the graph of Fig. 8, it is not difficult to find the invariant measure. For this purpose we only have to ascribe to every path of Fig. 8 the probability of its transition from one segment to another. If there is only one path, leaving the segment, the transition probability is equal to 1 - this is the case for all the paths except those that go out the segment d_1. There are several paths from d_1, and we have to distribute the unity probability among them. It is clear that the transition probabilities should be distributed proportionally to the lengths of the segments Δ_i. Then, using the formulae

$$\begin{aligned}
&\mu(d_i) = \mu(d_{i+1}) && \text{for } i = 1,2,3 \\
&\mu(d_i) = \mu(d_{i+1}) + \mu(\Delta_{i+1}) && \text{for } i = 4,5,6 \\
&\mu(d_7) = \mu(\Delta_8) && \\
&\mu(\Delta_5 + \Delta_6 + \Delta_7 + \Delta_8) = \mu(d_1), &&
\end{aligned} \tag{5}$$

we shall obtain the invariant probability distribution, shown in

Fig.7. It determines completely the statistics of the output signal. Let us, as an example, determine the distribution of numbers of maxima in successive impulses on the basis of this statistics.

As it is evident from Fig. 6, the number of maxima can change from 5 to 8. If, on completion of the preceding impulse, y emerges in the segment Δ_5, then there will be 5 maxima in the next impulse (the point will go along the segments d_4, d_3, d_2, d_1, get to one of Δ_i, and the impulse will be over), and so on. Therefore the probability for the impulse to have n maxima is equal to the conditional probability

$$\text{Prob (n maxima in the impulse)} = \frac{\mu(\Delta_n)}{\mu(\Delta_5+\Delta_6+\Delta_7+\Delta_8)} \qquad (6)$$

As it is easy to derive from Fig.6 and 7 this probability is proportional to the length of the segments Δ_n, that is, we might set approximately

$$\text{Prob (n maxima in the impulse)} \sim \sqrt{n-4}\cdot T^{-n} \qquad (7)$$

The relation (7) can be easily generalized for the case when the minimal number of maxima in the impulse is equal to n_0:

$$\text{Prob (n maxima in the impulse)} \sim \sqrt{n-n_0+1}\cdot T^{-n} \qquad (8)$$

To verify the relation (8), we experimentally constructed a histogram of the impulse distribution of the number of maxima for different parameters of the circuit of Fig. 1, that is, for different n_0 and T. It can be easily seen that the histograms of Fig. 8 are qualitatively compatible with the relation (8).

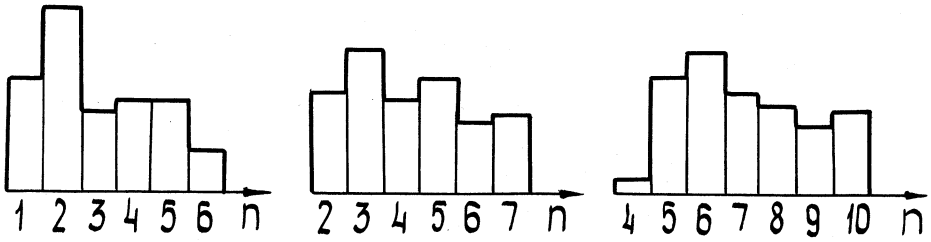

Figure 9
Histograms of the impulse distribution of the number of maxima

This method of obtaining the invariant measure and investigating properties of dynamical systems has been given the name of Markov partition (for more detail see [13,14]).

More complex regimes. We have just demonstrated the way of proving stochasticity of self-sustained oscillations of the generator of Fig. 1 and determining statistical properties of the output signal with the help of a number of idealizations. However, to describe more complex regimes, one should take into consideration finer details of the system dynamics.

As in the previous case, the main difficulties can be investigated in the framework of an one-dimensional Poincaré map, but this time the form of the map should differ from that of Fig. 4. Firstly, it should be noted that there is no discontinuity in the real system, and we have to connect the two parts of the mapping with a continuous curve. Secondly, the mapping is expanding not for all the values of the parameters; it may have a point with the zero slope: $d\bar{y}/dy = 0$. No the mapping will have the form, shown in Fig. 10, and its mathematical treatment is rather difficult.

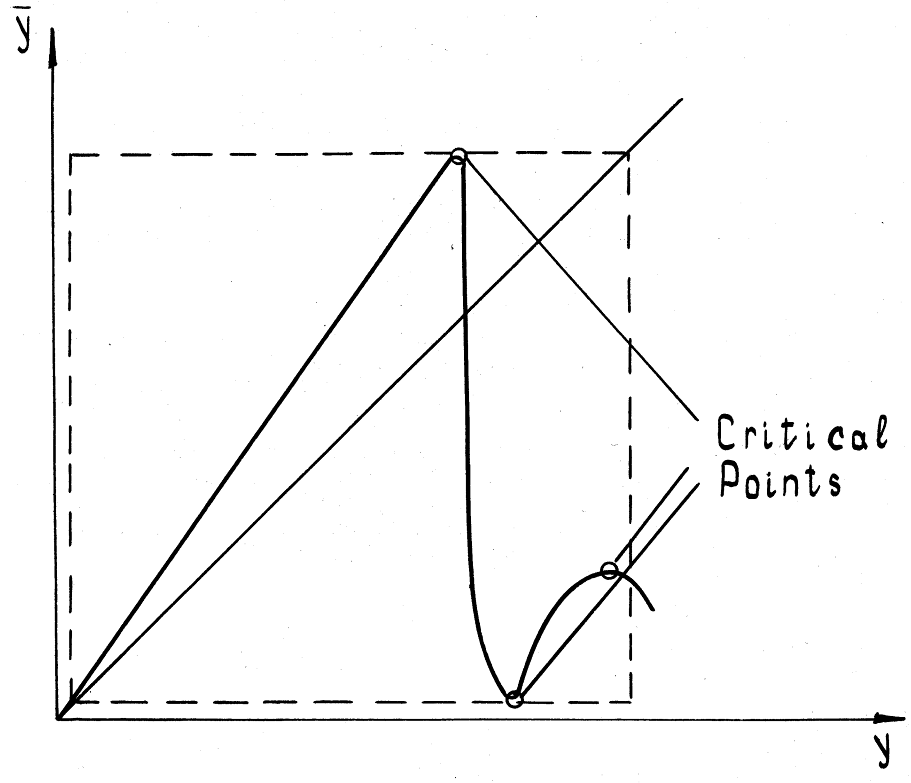

Figure 10
More realistic Poincaré map

First of all difficulties are due to the presence of critical points, where $d\bar{y}/dy = 0$. Stable periodic points appear to be possible, and stochasticity, which is not now attracting, is not observed. With changes in the parameters, the stable periodic points undergo complex bifurcations; sometimes the motion seems to be stochastic, but this stochasticity is rather the result of small random perturbations.

Complex periodic and stochastic states, observed in the simple noise generator, are depicted in Fig. 11. And here analogous regimes, observed in a homogeneous Belousov-Zhabotinsky reaction, are given for comparison [15].

The resemblance in the behaviour of these two completely different systems is not difficult to account for. The thing is that in a chemical reaction there also works a switching mechanism for chaos, and the dynamics of the reaction components are described by equations, similar to the system (2). Let us note that the switching mechanism for chaos in application to chemical kinetics was discovered for the first time by Roessler [16].

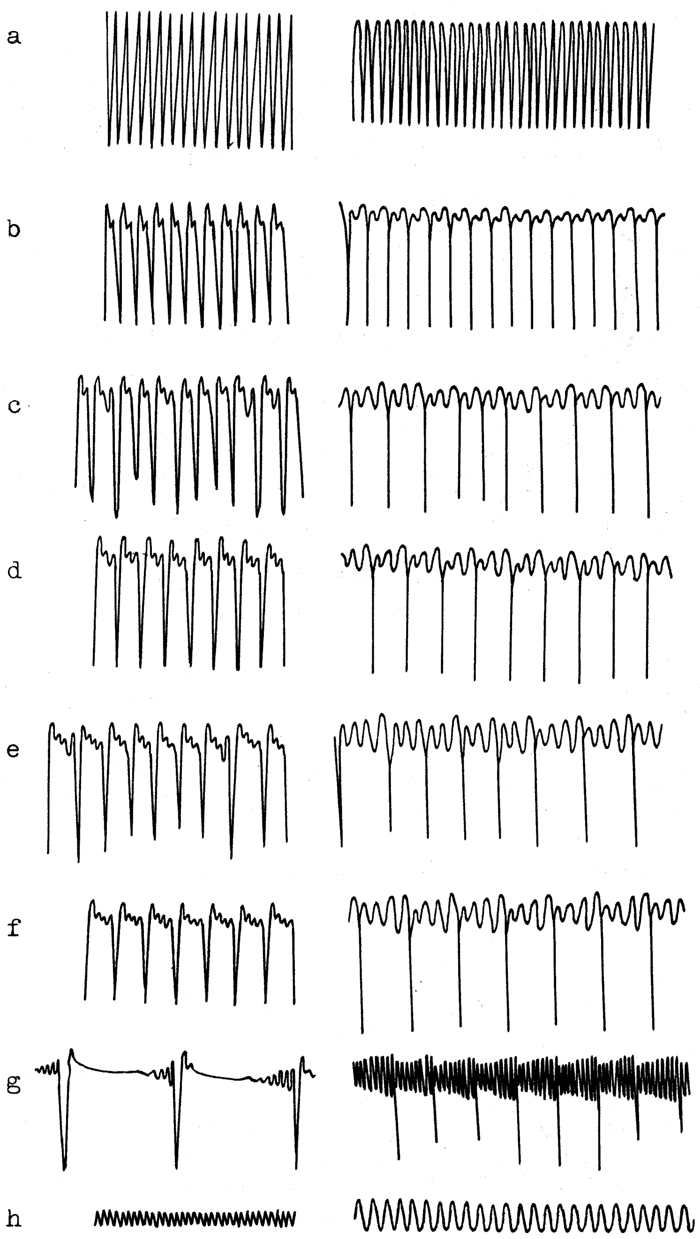

Figure 11
Multiple-peak periodic (a, b, d, f and h) and stochastic (c, e, g) iscillations in the Belousov--Zhabotinsky reaction (left column, the characteristic duration of the period is about 10 min) and in the simple electronic noise generator (right column, the characteristic duration of the period is about msec)

Generalizations. The switching mechanism for chaos can be presented schematically in the following way

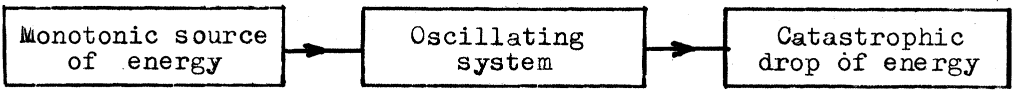

Figure 12

The above-considered circuit (see Fig. 1) gives one of the realizations of such a mechanism. Here the source of energy is a vacuum tube, the oscillating system is a LC circuit, the switching drop of energy is effected by a tunnel diode. It is clear that a concrete realization of the blocks of Fig. 12 might be different. In Fig. 13a a negative resistance is used instead of the vacuum tube [8]. Such an alteration does not lead to any qualitative change both in the mathematical model - the equations (2) - and in the real working regimes of the generator. In the modification of Fig. 13b the source of energy is changed. For this purpose a pump (external oscillations with double frequency) is used. It excites parametrically the oscillations of the main frequency. In this case the mathematical model is more complex - the number of equations remains the same (three), but they are not autonomous any more.

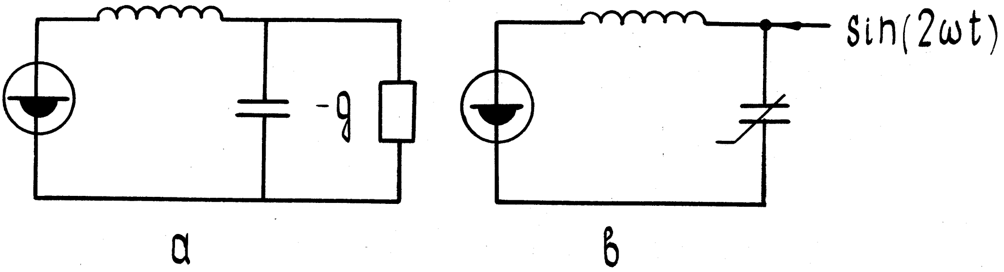

Figure 13

OTHER DISSIPATIVE MECHANISMS

The scheme of Fig. 12, illustrating the mechanism for chaos appearance due to nonlinear drop of energy, can be generalized still further. For example, the drop of energy to a thermostat can be effected not only directly, as in Fig. 12, but also as in Fig. 14.

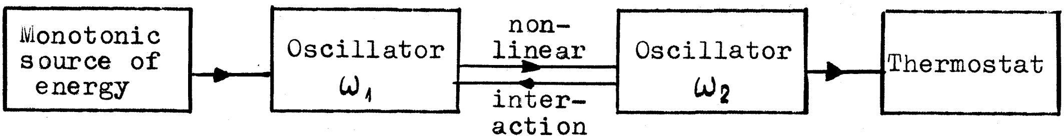

Figure 14

The mean energy flux is directed from ω_1 to ω_2, while in the absence of source and sink there is no flux. As an example, let us consider the interaction of oscillators with multiple frequencies ω_1 and $\omega_2 \approx \omega_{1}/2$ (ω_1 decays into $\omega_2 + \omega_2 - \delta + \alpha$, where δ is linear, and α is nonlinear frequency deviation from pure resonance), when the main oscillation possesses an increment, and the subgarmonic damps [17]. When damping, increment and nonlinearity are small, the averaged equations for amplitudes and phases of oscillations for frequences ω_1 and ω_2 are used as the initial ones

$$\begin{aligned} \dot{A}_1 &= A_2^2 \operatorname{Sin}\Phi + \gamma A_1 \\ \dot{A}_2 &= -A_1 A_2 \operatorname{Sin}\Phi - \nu A_2 \\ \dot{\Phi} &= \left(\frac{A_2^2}{A_1} - 2A_1\right) \operatorname{Cos}\Phi + \alpha A_2^2 + \delta \end{aligned} \tag{9}$$

Here $\Phi = \varphi_1 - 2\varphi_2$ is the phase difference of oscillators, regulating the energy exchange among them. In the conservative case ($\gamma = \nu = 0$) this system is fully integrable and describes only simple (static and periodic) motions. If the source (γ) and the sink (ν) are present, the dynamics can be stochastic, as is clear from the numerical and analog simulation. The phase portrait of the attractor for the system (9) with $X = -A_1 \operatorname{Sin}\Phi$, $Y = A_1 \operatorname{Cos}\Phi$, $Z = A_2^2$ is shown in Fig. 15.

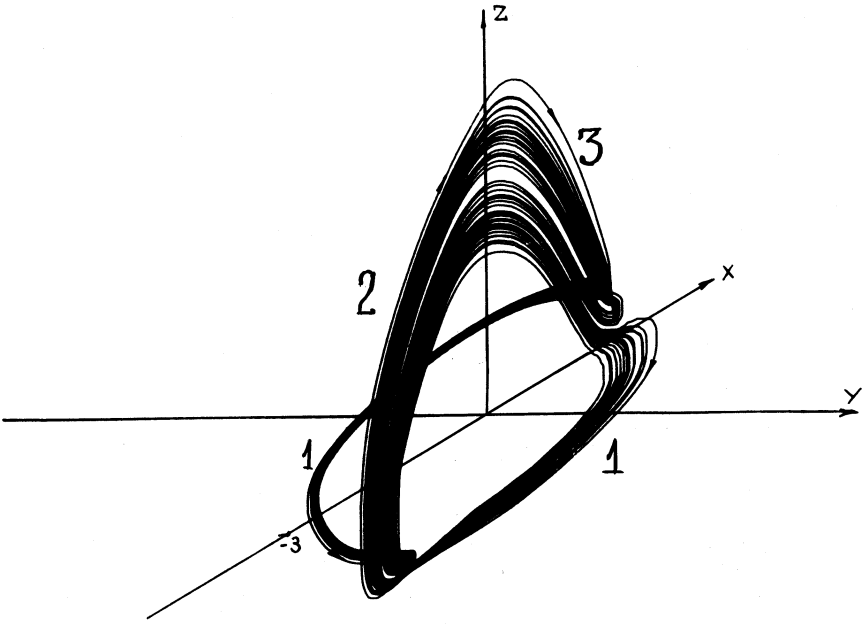

Figure 15
Phase portrait of the strange attractor for the system (9)

It is obtained on an electronic analog integrator for $\gamma/\nu = 0.25$, $\delta = 1.5$, $\alpha =$. In this case there are no sharp boundaries dividing the parts of the trajectories into exponentially increasing and returning ones, the latter describing the drop of energy.

Nevertheless, such parts can be singled out in a sufficiently unique way. In part 1 the oscillations increase almost exponentially in the oscillator ω_1, then the energy is transmitted to the subgarmonic (part 2) and, at last, in part 3 the drop of energy (dissipation) prevails, that is, the system returns to the vicinity of the initial state. The physical picture resembles the above-considered, but for the switches. This mechanism can be called decay.

In a likewise manner chaos appears in the resonance interaction of three oscillators ω_1, ω_2 and ω_3: $2\omega_1 = \omega_2 + \omega_3$, if the oscillator ω_1 is connected with sources of energy, and the oscillators-satellites ω_2 and ω_3 transmit the energy to a thermostat (damping). The interaction of this kind, which is usually called four-quantum decay, is widespread in physics. It is connected, in particular, with the modulation instability of Stokes water waves, Langmuir waves in a plasma and so on. With sources and dissipation missing, modulation instability results in the growth of satellites, which afterwards return the energy to the main oscillator, and the process appears to be periodic. And in the nonconservative case, when characteristic periods of instability, dissipation and nonlinear interaction are of the same order, stochastic self-modulation might be established. When observed, it is characterized by a continuous power spectrum and a dropping correlation function [18]. The mechanism for the appearance of modulation chaos is also illustrated by Fig. 12.

However, in many cases the processes of energy supply (that is, instability) and dissipation (that is, returning), which are necessary for the appearance of dissipative chaos, cannot be separated in such a trivial way as in Fig. 12 and 14. For example, the physical mechanism for chaos is quite different in the parametrical excitation of damping oscillators ω_1 and ω_2, taking part in the decay interaction of the type $\omega_1 \rightarrow \omega_2 + \omega_3$. For the complex amplitudes ofoscillators we have the system [19]

$$
\begin{aligned}
\dot{a}_1 &= ha_2^* - \nu_1 a_1 - a_2 a_3 \\
\dot{a}_2 &= ha_1^* - \nu_2 a_2 + a_1 a_3^* \\
\dot{a}_3 &= \qquad - \nu_3 a_3 + a_1 a_2^*
\end{aligned}
\qquad (10)
$$

Due to the syncronization of the phases of the oscillators ($\arg a_1 + \arg a_2 \rightarrow 0$, $\arg a_1 - \arg a_2 - \arg a_3 \rightarrow 0$ for $t \rightarrow \infty$) the amplitudes in (13) can be considered purely real. Then this system transforms into the Lorenz system with the only difference that in the equation for a_2 there is the additional item $a_1 a_3$. This difference, however, is of no significance, and the dynamics of (10) are analogous to those of the Lorenz system [19]. Chaos in these systems is also dissipative, for they exhibit only simple motions for $h = \nu = 0$. However, the signs and absolute values of the oscillation amplitudes govern here not the drop of energy to a thermostat, but its supply to the system. This is an absolutely different mechanism for dissipative chaos - a "parametrical" one. It is also widespread in simple systems.

CONCLUSION

We have considered two groups of essentially dissipative mechanisms for chaos. Other mechanisms are also possible. One of them, for example, consists in the stochastization of coupled nonlinear oscillators [20,21], nonlinearity of which is essentially dissipative. One may hope that the growth of the number of new examples of stochastic behaviour in dissipative systems will enable us to single out new mechanisms for chaos and, hence, to construct as complete theory of stochastic self-oscillations as we have for periodic oscillations. Physical mechanisms for the appearance of stochasticity should play an important role in this theory, since they can help to work out quantitative criteria for stochasticity, for example, such as the Chirikov criterion (overlapping of resonances) for the stochastization of Hamiltonian systems.

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