Self-Exciting Oscillator for the Radio-Frequency Range with Stochastic Behavior

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A self-exciting oscillator for radio-engineering applications is discussed which differs from an ordinary oscillator with the tuned circuit in the grid branch by the introduction of a tunnel diode connected in series with the circuit inductance. The dynamic equations describing the network are qualitatively and numerically analyzed. The structure of the attracting region in the phase space (of the attractor) is investigated by reduction to a one-dimensional point representation whose stretching nature ensures stochasticity of the generated oscillations. Experimentally observed realizations and spectra of the noise signal are presented.

INTRODUCTION

Most recently, new objects of investigation appeared in radio engineering which are attractive from the theoretical and practical standpoint, namely, self-excited noise generators [1, 2] in which the output signal statistics is determined by complex dynamics of a system without noise sources rather than by amplification of fluctuations. In contrast to generators of periodic oscillations (of the sinusoidal or relaxation types [3]), such a self-exciting oscillator built even according to a very simple schematic [2, 4] delivers to the load a signal exhibiting all the characteristics of a random signal (a continuous spectrum, decrease in autocorrelation, etc.). Appearance of simple noise generators, i.e., stochastic self-exciting oscillators, is associated with recent advances in the theory of nonlinear oscillations of systems with a number of degrees of freedom greater than two.

The fact of the matter is that some time ago we knew only one mathematical portrayal of self-exciting oscillations, i.e., the limit cycle, which was just satisfactory for the generation of periodic signals. Any more complex patterns in the phase space of self-exciting oscillatory systems appeared to be unstable and, therefore, physically unrealizable. However, towards the end of 1960s mathematicians determined that in addition to simple stable patterns (attractors) there can also exist complex patterns to which corresponds the stochastic behavior of a dynamic system, i.e., random signal generation [5, 6]. Thus appeared the mathematical portrayal of stochastic self-excited oscillations, i.e., the "unusual attractor" [7]*. The phrase "stochastic behavior of a dynamic system" must be understood in the sense that though exact specification of the initial point in the phase space completely defines the subsequent trajectory, this trajectory can be very complex and practically indistinguishable from a random process [8, 9, 10]. For clarity, we can refer to the analogy with generators of random numbers used in computers: although operations performed in computers are deterministic, the generated sequence of numbers does not differ from a random sequence.

After the discovery of strange or stochastic attractors, attempts were made practically at once to describe, with their help, the occurrence of hydrodynamic turbulence [7, 11]. Marked advances were now accomplished in this direction that are mainly associated with the investigation of the so-called Lorenz's system [12] which is a maximally simple model of a free-convection turbulence. Detailed numerical calculations and qualitative considerations based on bifurcation theory made it possible to conclude with enough certainty that in Lorenz's system within a wide interval of variation in parameters there are no other attractors except the stochastic attractor [13, 14]. A few more, simple enough systems are known where stochasticity was determined by numerical analysis. Such systems describe the kinetics of chemical reactions [15], operation of a dynamo [16], nonlinear interaction between waves [10, 17].

From the standpoint of history, it turned out that in the development of the concepts concerning the strange attractors and stochastic self-excited oscillations the classical application of the theory of dynamic systems, i.e., radio engineering, was circumvented. The concept of constructing noise generators for radio engineering that are based on the stochastic attractor was for the first time proposed only four years before [15] although a greater part of the known systems with stochastic attractors (for example, Lorenz's system) turned out to be simple enough to simulate it using an analog computer and thereby obtain an actual noise generator. Let us also note the experiments in which noise was observed in distributed systems for the radio range, i.e., LC lines [19]. It is possible that the origin of the observed noise was due to a stochastic process.

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*We shall also use the term "stochastic attractor" proposed by Ya. G. Sinai [8].

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In this work we present the results of a theoretical, numerical and experimental investigation of one of the simplest self-excited noise generators, i.e., a relaxation oscillator similar to the one in [4].

1. CIRCUIT OPERATION AND EXPERIMENTS

Let us discuss an oscillator built according to the schematic in Fig. 1a. From the classical self-excited oscillator with the tuned circuit in the grid circuit the oscillator differs only by the presence of a tunnel diode connected in series with an inductance. The operation of the oscillator is described by the following equations:

$$\begin{aligned} C\dot{U} &= -I, \\ LC\dot{I} &= (MS - rC)I + C(U - V), \\ C_1\dot{V} &= I - I_{td}(V). \end{aligned} \qquad (1)$$

Here C_1 is capacitance of the tunnel diode, S is the mutual conductance of the tube, M is coefficient of mutual inductance. In discussion of the oscillator operation we shall assume that the tube characteristic is linear. This is justifiable by the fact that under operating conditions of interest oscillations are limited by the nonlinear characteristic of the tunnel diode $I_{td}(V)$ (Fig. 1b) to such a level that nonlinearity of the tube does not show up.

The operation of the oscillator can be qualitatively described as follows. Until current I and voltage U are small, the tunnel diode has no substantial effect on the oscillations in the tuned circuit and the oscillations do not increase as a result of the energy delivered by the tube. Through the tunnel diode then flows current I and the voltage on the diode is determined by branch α of the characteristic $I_{td}(V)$. However, when current I attains value I_m, almost instantaneous switching of the tunnel diode occurs (the switching speed depends on how small capacitance C_1 is, i.e., voltage V_m is established in a jump). The current through the tunnel diode then decreases and undergoes inverse switching from section β onto section α. In result of two switchings the tunnel diode almost completely "consumes" the energy that entered the tuned circuit and oscillations begin to increase again.

The generated signal U(t) represents therefore a sequence of trains of growing oscillations and the end of each train is accompanied by voltage pulse V(t). Obviously, it is not clear from the above description whether the steady-state regime will be periodic or stochastic. This can be ascertained by further investigation of Eqs. (1), but we shall postpone this until the following section and present experimental results below.

The schematic shown in Fig. 1 was implemented using one-half of the 6N1P triode (E_a = 250 V, r_c = 30 ohms). The tuned circuit was made up of capacitance C = 1.5 μf and the inductance L = 5.7 mh. Four 3I306G tunnel diodes ($I_m \approx 7.25$ mA, $V_m \approx 1.08$ V, $C_1 \approx 30$ nF) were connected in parallel in the tuned circuit. In this case, the dimensionless parameters (see further on Eqs. (2)) were equal to $g \approx 2.4$, $\varepsilon \approx 4.8 \cdot 10^{-5}$. It was convenient to change the increment in increase in the oscillations in the tuned circuit, i.e., the value of h, by changing resistance r. The smallest attainable losses in the tuned circuit, as determined by the elements of the network alone, were equal to $r_0 \approx 8.2$ ohms.

For $R = r - r_0 \approx 14.5$ ohms purely periodic oscillations were excited in the tuned circuit and they were limited by the nonlinearity of the tube at such a low level that the diodes did not switch ($I < I_m$). For $R \approx 13.5$ ohms the amplitude of the oscillations reached its threshold value and signal U(t) represented long bursts of oscillations which were rarely interupted by the switchings of the diodes. Only for R > 11 ohms the triode nonlinearity had no effect, i.e., signal was generated in the form of long trains inside each oscillations exponentially were increasing and the transition from one train to another was

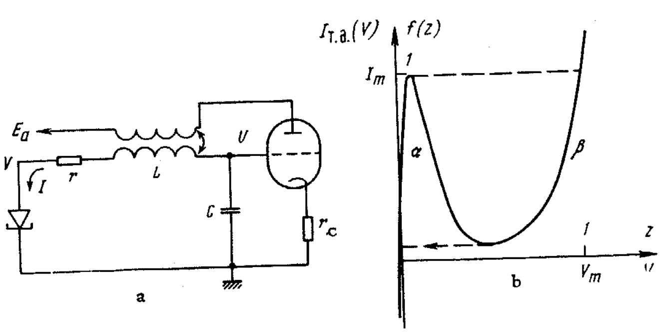

Fig. 1. a) self-excited noise generator circuit, b) the voltage-current characteristic of the tunnel diode.

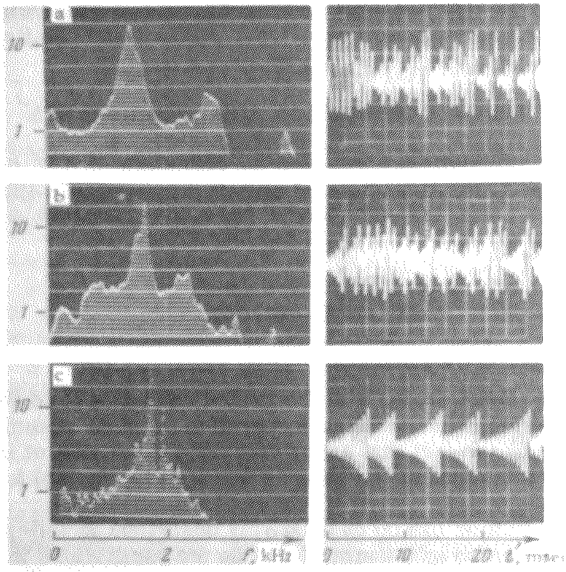

Fig. 2. Scope traces of the output signal U(t) (to the right) and its spectra in the logarithmic scale (to the left) for different Q-factor of the tuned circuit (in the upper figure the value of R is minimum).

accompanied by a voltage pulse on the tunnel diode V(t). For no value of $R < 11$ ohms were we able to observe periodic regime, i.e., a random signal with a continuous spectrum was generated. It is evident from the spectra and scope traces shown in Fig. 2 how with decreasing R the increment of oscillations h increases and the mean length of a train decreases; the peaks at the repetition frequencies of the trains in the spectrum become then increasingly smoother. A greater part of the energy is contained in the main maximum corresponding to the oscillation frequency of the tuned circuit.

2. MATHEMATICAL ANALYSIS OF THE MODEL

Analyzing Eqs. (1), let us change to dimensionless variables $x = I/I_m$, $z = V/V_m$, $y = UC^{1/2}/(I_m L^{1/2})$, $\tau = t(LC)^{-1/2}$. We obtain

$$\begin{aligned} \dot{x} &= 2hx + y - gz, \\ \dot{y} &= -x, \\ \varepsilon \dot{z} &= x - f(z). \end{aligned} \qquad (2)$$

Here $h = 0.5(MS - rC)(LC)^{-1/2}$ is the increment in increase of oscillations in the tuned circuit in the absence of the diode; $g = V_m C^{1/2}/(I_m L^{1/2})$ is a parameter defining the degree of influence of the tunnel diode on the processes in the tuned circuit: $\varepsilon = gC_1/C$ is a small parameter which is proportional to the tunnel diode capacitance; $f(z) = I_{td}(V_m z)/I_m$ is normalized diode characteristic (see Fig. 1b).

The system of Eqs. (2) has small parameter ε at the derivative and therefore all motions in the phase space (Fig. 3) can be divided into fast motions, i.e., switchings of the diode (straight lines x = const, y = const) and slow motions, i.e., oscillations in the presence of which the diode voltage follows the current (the corresponding trajectories lie on surfaces A and B ($x = f(x)$, $f'(z) = 0$) corresponding to segments and of the diode characteristic [20].

Approximate shape of the phase space of the system of Eqs. (2) is shown in Fig. 3. The system of equations has one unstable (for $2h > g/f'(0)$) state of equilibrium $x = y = z = 0$. The trajectories lying on surface A untwist themselves about the unstable focus and eventually reach the edge of surface A. Here the tracer point suffers a discontinuity along the line of fast motions onto surface B. Passing over B the tracer point jumps back onto surface A and falls into the vicinity of the state of equilibrium, i.e., a new train of increasing oscillations is initiated.

Further analysis will be substantially simplified when we pass from a description of trajectories that is continuous in time to a description that is discrete in time. In fact, we shall note only those

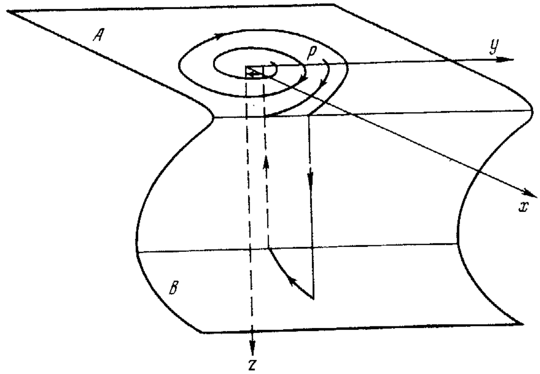

Fig. 3. Phase space of the system of Eqs. (2).

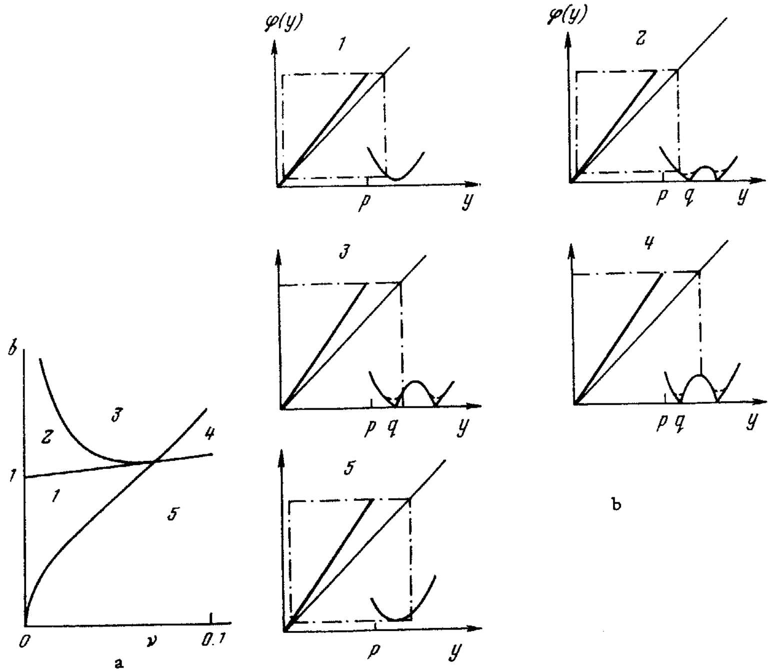

Fig. 4. a) division of the plane of parameters b and ν into regions of different behavior of T; b) mappings of T which are obtained using the piecewise-linear approximation of f(z).

point of a trajectory in which y (i.e., the output voltage U) attains a maximum, i.e., let us represent succession T: $y_i \rightarrow y_{i+1} = \varphi(y_i)$ by semi-straight line $\Sigma(x = z = 0)$, $y > 0$) in itself. All the trajectories beginning and ending on Σ can be divided into the two classes: 1) those which entirely lie on surface A, i.e., they make a turn around the state of equilibrium and 2) those reaching surface B. Those two groups of trajectories are divided by trajectory P which approaches the break-off line along the tangent trajectory.

Representation of φ(y) can be analytically expressed only when f(z) is approximated by a piecewise-linear function. Let us set, for example, $f(z) = z/\varkappa$ for $z<\varkappa$, $f(z)=(1-\varkappa-z)/(1-2\varkappa)$ for $\varkappa<z<1-\varkappa$ and $f(z)=(z-1+\varkappa)$ for $1-\varkappa<z$. The equations of slow motions will then become linear

$$\dot{x}=2\nu x+y+k, \quad \dot{y}=-x, \tag{3}$$

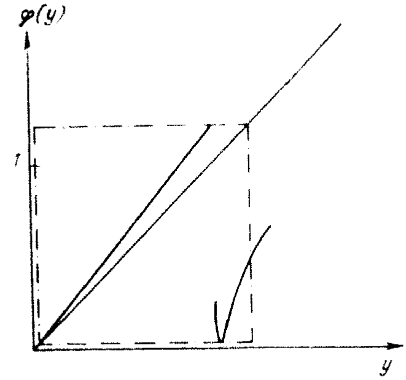

Fig. 5. Mapping $y \rightarrow \varphi(y)$, constructed numerically for the system of Eqs. (2) for $h = 0.074$, $g = 2.8$, $\varepsilon = 0.004$. The attractor is shown by the dash-dot block.

where $k = 0$ on A (now already A and B are planes) and $k = b = g/(1-\varkappa)$ on plane B, $\nu = h - 0.5\varkappa g$. By joir ing the solutions of (3) in a standard manner on planes A and B [3] we can readily obtain an expression $\varphi(y)$ which is similar to that in [4, 18]. Without writing unwieldy formulas, let us present only the divi sion of the range of parameters b and ν into different types of behavior of $\varphi(y)$ (Fig. 4).

The representations in Fig. 4b have a point of inflection q together with discontinuity p (correspo ing to trajectory P). The appearance of singularity at point q is related to the fact that for a given choice of f(x) (f(z) = 0 for $x = 1-\varkappa$) the state of equilibrium lies exactly under the line of discontinuity. When we take into account the fact that $f(z) \neq 0$, the inflection will disappear as shown by the dotted cur in Fig. 4b. In all the representations in Fig. 4b there is an attracting region into which all the trajectories enter. Inside this region there occur at first a number of iterations with $y_i < p$ (a growth of oscillations in a train corresponds to this situation) and then iteration with $y_i > p$ returns the represent: point again onto the linear segment (one train is replaced by another).

For small values of ν the following condition is satisfied inside the attracting region $|\varphi'| > 1$, i. the mapping is a stretching one — in successive iterations two close points diverge. The divergence o: close trajectories is one of the traits of stochasticity [8, 10] and it ensures at once that stable limiting cycles will not exist. In this case the mapping has an invariant ergodic measure with regard to which the mapping is a mixing one (this follows from the results of [14]).

However, conclusions reached when using the piecewise-linear approximation of f(z) can turn ou to be invalid since in such description the behavior of the trajectories close to P is incorrectly present We have therefore constructed the mapping of T numerically. The values of the parameters were sele as follows: $h = 0.074$; $q = 2.8$; $\varepsilon = 0.004$; the characteristic f(z) was approximated by the function $f(z) = z \exp(3.61 - 13.5z) + \exp(6.5(z-1)) - \exp(-6.5)$. For integration we used the BESM-6 computer ar the Runge-Kutta method with a step of $2 \cdot 10^{-4}$. The obtained function $\varphi(y)$ is shown in Fig. 5. A critic point in which $\varphi' = 0$ appeared now close to P, but it was not possible to determine accurately the disc tinuity region. Apparently, mapping of T is in fact continuous, but $\varphi' \sim \exp(\varepsilon^{-1})$ (!), i.e., the assum tion that the piecewise-linear approximation describes the true situation sufficiently closely was confirmed. And yet the existence of critical points inside the attracting region degrades the stochasticity the system.

As it follows from [21, 22], the mappings with critical points have a stable limiting cycle for almost all values of the parameters. However, firstly, a stable cycle is surrounded by a stochastic nonattracting area [23]; secondly, the cycle period can be very large and over large time intervals the realization will appear to be random, thirdly, the region of the space of parameters in which a given cycle is stable is, as a rule, very small. In numerical experiments [24] the mappings with critical points demonstrate therefore a statistical behavior; however, here the term "complex dynamics" [18] would be possibly more suitable. And although the stochasticity then observed is the result of small noise (for example, of errors in the environment), the statistical properties of the signal are determined, judging by everything, by the intrinsic dynamics of the system rather than by the statistics of the noise.

The system of Eqs. (2) demonstrates therefore a behavior that is indistinguishable from a stochastic behavior and the signal obtained in the self-excited oscillator is a random signal.

In the self-excited noise generator here described, only one of the possible mechanisms causing stochasticity in self-exciting oscillatory systems is realized. Other mechanisms, like decaying, associated with inertia of nonlinearity and others [10], can be realized in radio systems. In connectio with investigation of self-excited noise generators proper, the problems concerning the effect of an external signal on them, the interaction of a number of such generators and many others, are of obvious interest. Such investigations are just being started.

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