DAMOCO Toolbox

Brief illustration to the theory

Synchronization and its quantification

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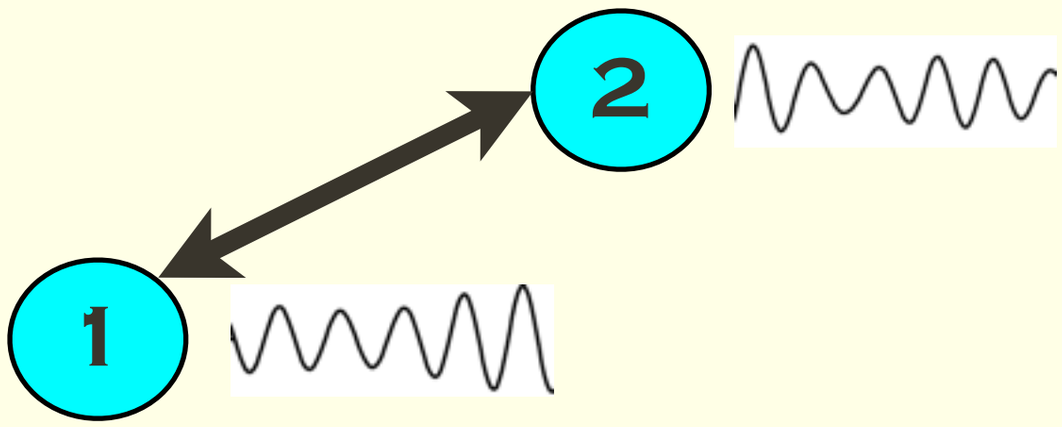
Synchronization

Synchronization is adjustment of rhythms of self-sustained oscillators due to their interaction. It is manifested via

- phase locking;
- frequency locking.

Two oscillators



Frequency locking:

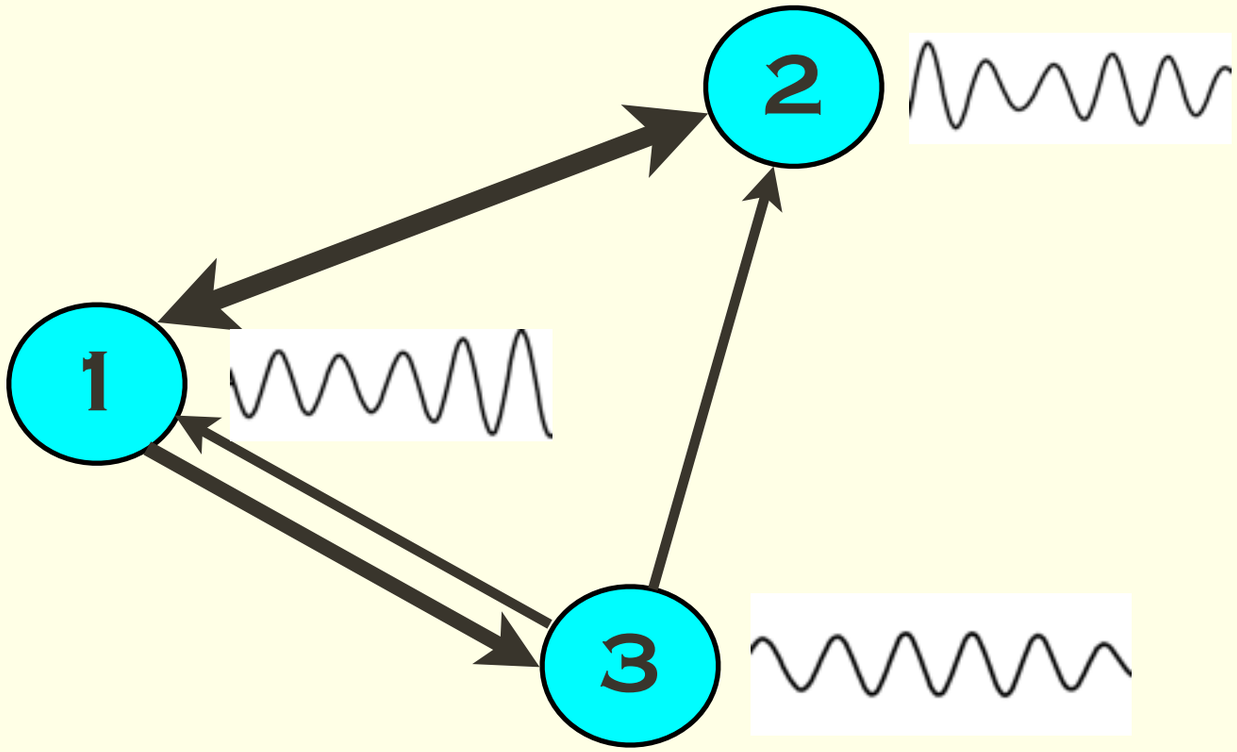
$$n\Omega_1 - m\Omega_2 = 0$$

Phase locking:

$$|n\varphi_1 - m\varphi_2| < \text{const}$$

Here ***n,m*** are positive integers

Phase and frequency locking: three oscillators



Frequency locking:

$$n\Omega_1 + m\Omega_2 + l\Omega_3 = 0$$

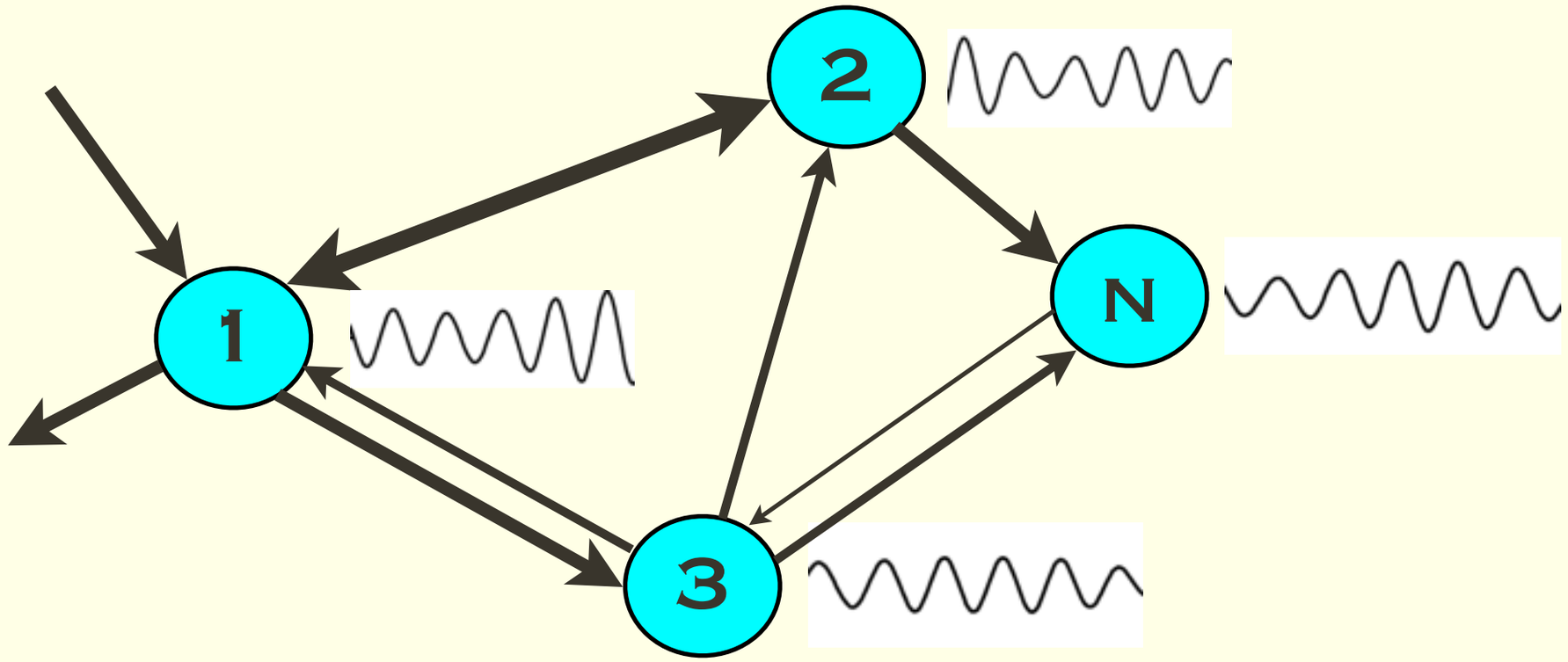
Phase locking:

$$|n\varphi_1 + m\varphi_2 + l\varphi_3| < \text{const}$$

Here ***n,m,l*** are integers which can be both positive and negative

For noisy systems the locking condition is fulfilled only approximately!

Phase and frequency locking: *N* oscillators



- For each pair we can test for phase locking from data
- For each triplet we can test for 3 pairwise locking conditions; if they are not fulfilled, we can test for triplet locking
- Locking conditions for ***N>3*** can be formulated similarly; however such high-order locking is unlikely

Synchronization index

(also known as phase locking value)

- Pairwise index: $\gamma_{n,m} = \left| \langle e^{i(n\varphi_1 - m\varphi_2)} \rangle \right|$

- Triplet index: $\gamma_{n,m,l} = \left| \langle e^{i(n\varphi_1 + m\varphi_2 + l\varphi_3)} \rangle \right|$

Notice: generally, $0 \leq \gamma_{n,m} \leq 1$, $0 \leq \gamma_{n,m,l} \leq 1$

Notice: generally, the index quantifies the interaction between the systems or correlation between the signals.