DAMOCO Toolbox

Brief illustration to the theory

Protophase-to-phase transformation: why is it important?

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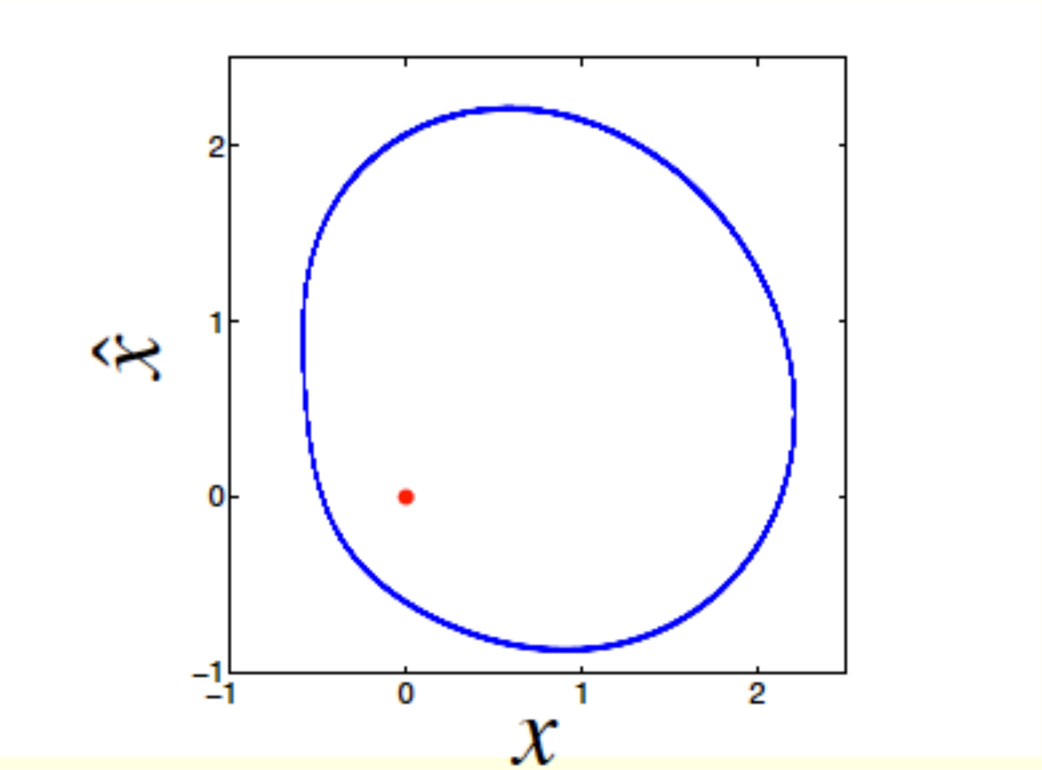
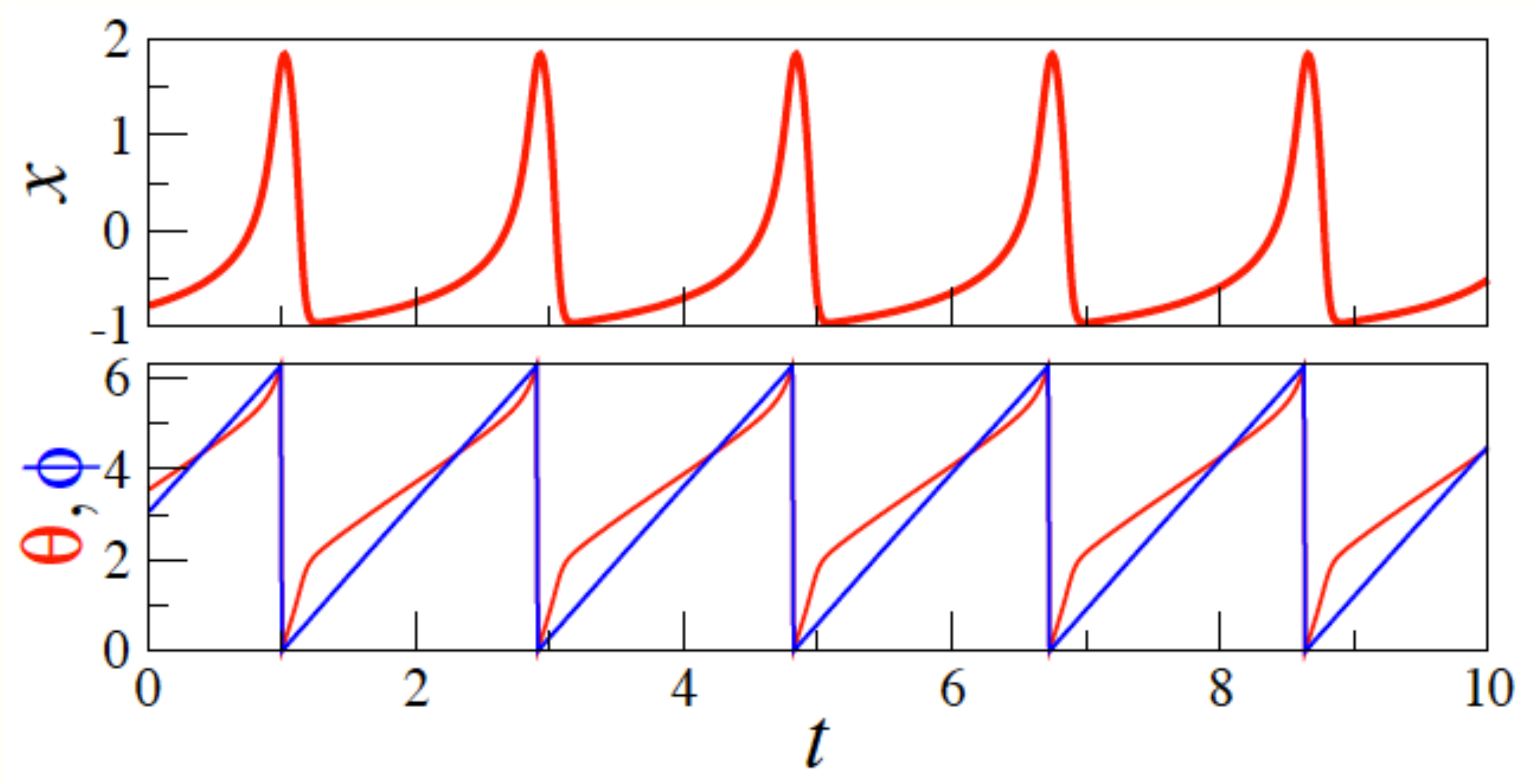
From time series to phase in two steps

1. From an oscillatory signal a protophase can be obtained, e.g., via the Hilbert transform, complex wavelet transform, etc
2. Phase is obtained from the protophase by means of the transformation $\theta \rightarrow \phi(\theta)$

Example: Hindmarsh-Rose neuronal oscillator

$$
\begin{aligned}
\dot{x} &= y - x^3 + 3x^2 - z + I\,, \quad I = 5.1 \\
\dot{y} &= 1 - 5x^2 - y \\
\dot{z} &= 0.006 \cdot (4(x + 1.56) - z)
\end{aligned}
$$





$$\theta = \arctan(\hat{x}/x)$$

$$\phi = \omega_0 \int_0^{\theta} \left[\frac{d\theta}{dt}\right]^{-1} d\theta$$

Why is the transformation $\theta \to \phi$ important?

Example: synchronization index

Data from two nonidentical and uncoupled Hindmarsh-Rose neurons with $I_1 = 5, I_2 = 5.1$

Synchronization index from protophases $\gamma = \left| \langle e^{i(\theta_1 - \theta_2)} \rangle \right| \approx 0.13$

Synchronization index from phases $\gamma = \left| \langle e^{i(\phi_1 - \phi_2)} \rangle \right| \approx 0.02$

Phase equations for two coupled oscillators

$$\frac{d\varphi_{1,2}}{dt} = \omega_{1,2} + h_{1,2}(\varphi_{1,2}, \varphi_{2,1})$$

- Can be reconstructed either from phases or from protophases

Why is the transformation $\theta \rightarrow \varphi$ important?

True equation in terms of phases
$$\frac{d\varphi_1}{dt} = \omega_1 + q_1(\varphi_1, \varphi_2)$$

Equation in terms of protophases

small term

$$\frac{d\theta_1}{dt} = \omega_1 + f(\theta) + \hat{q}_1(\theta_1, \theta_2)$$

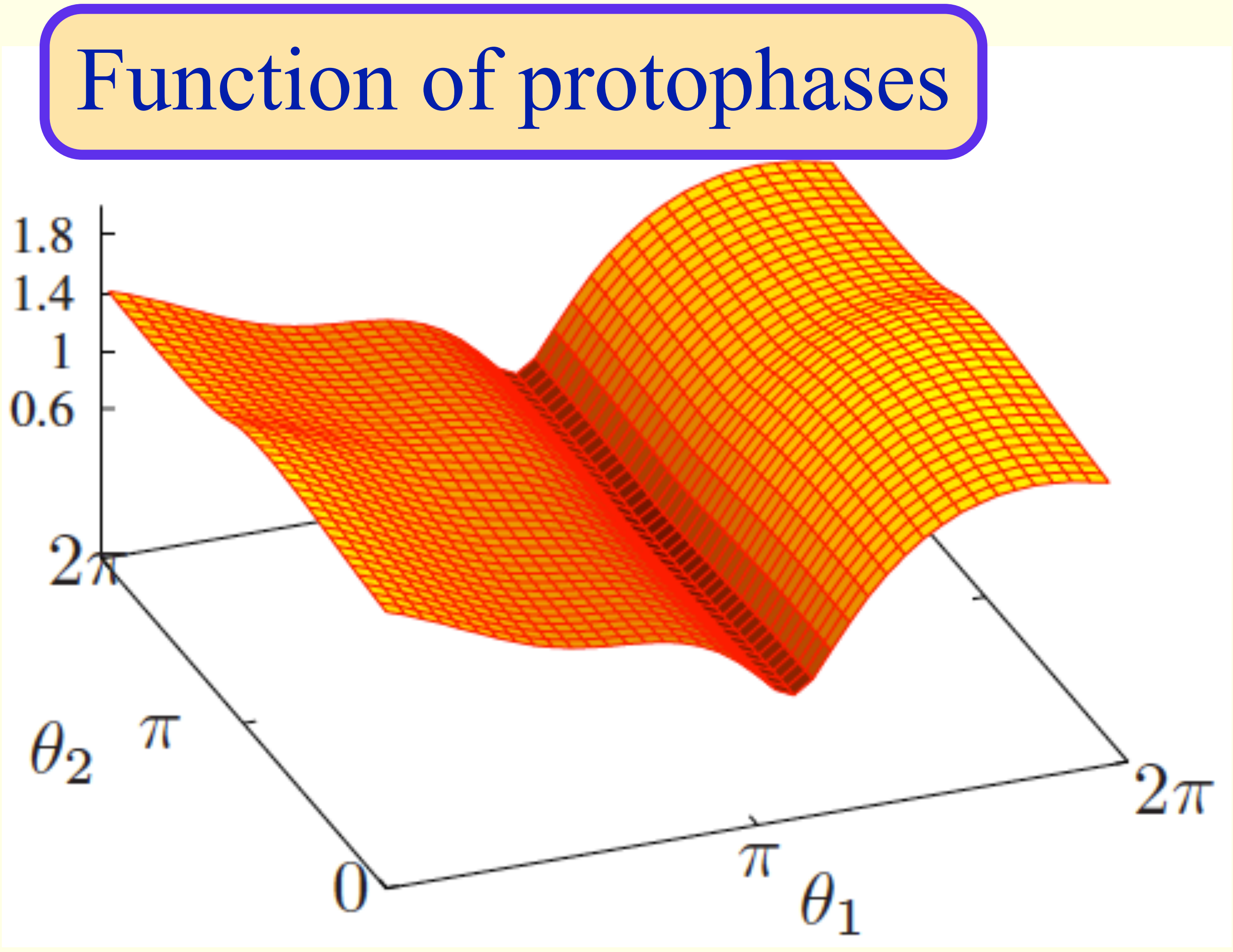
1. Generally not small
2. Has no physical meaning
3. Masks the term we need!

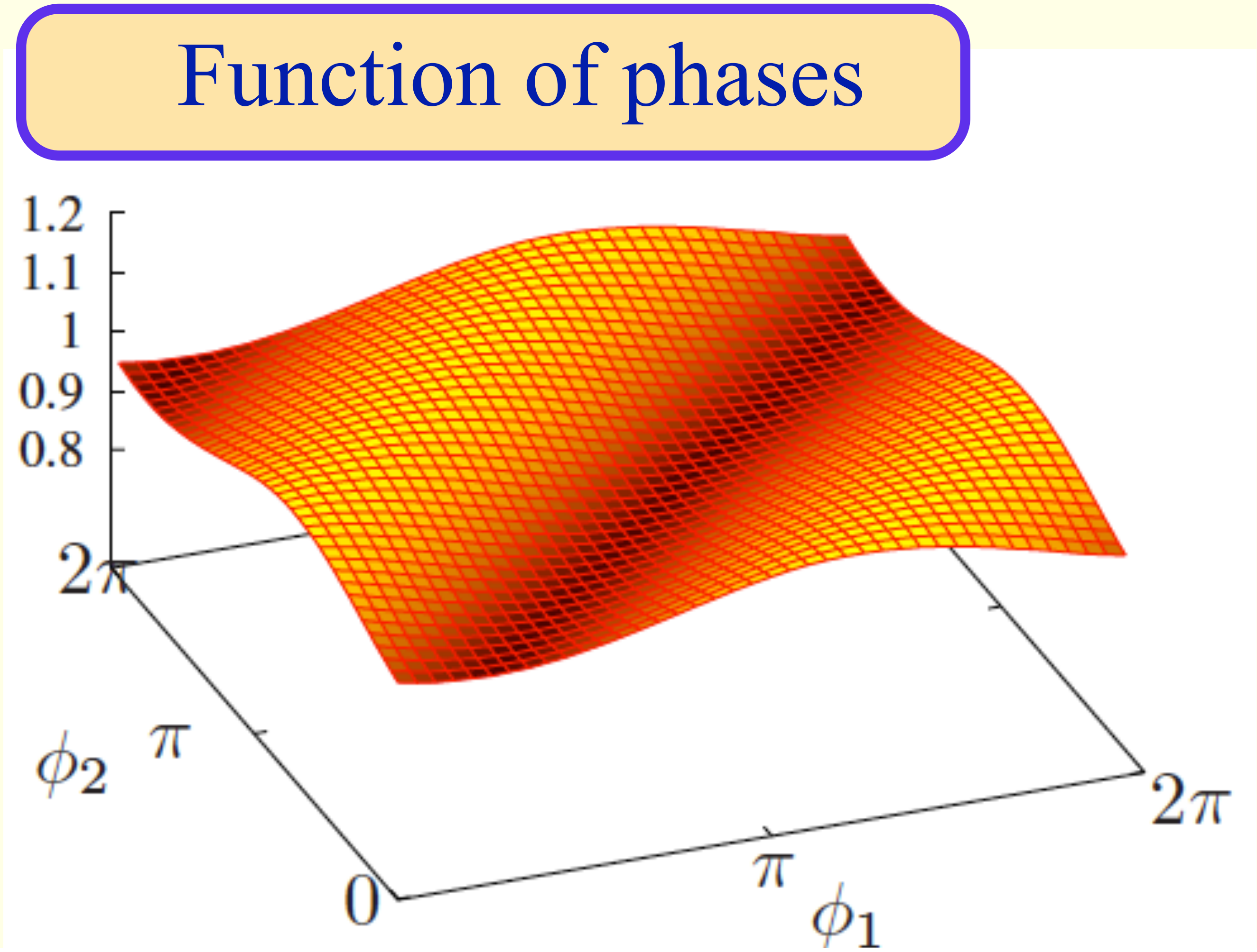
Why is the transformation $\theta \to \phi$ important?

Simple, but illustrative example

$$\dot{\phi}_1 = \omega_1 + \varepsilon_1 \sin(\phi_2 - \phi_1 - \beta_1)$$
$$\dot{\phi}_2 = \omega_2 + \varepsilon_2 \sin(\phi_1 - \phi_2 - \beta_2)$$

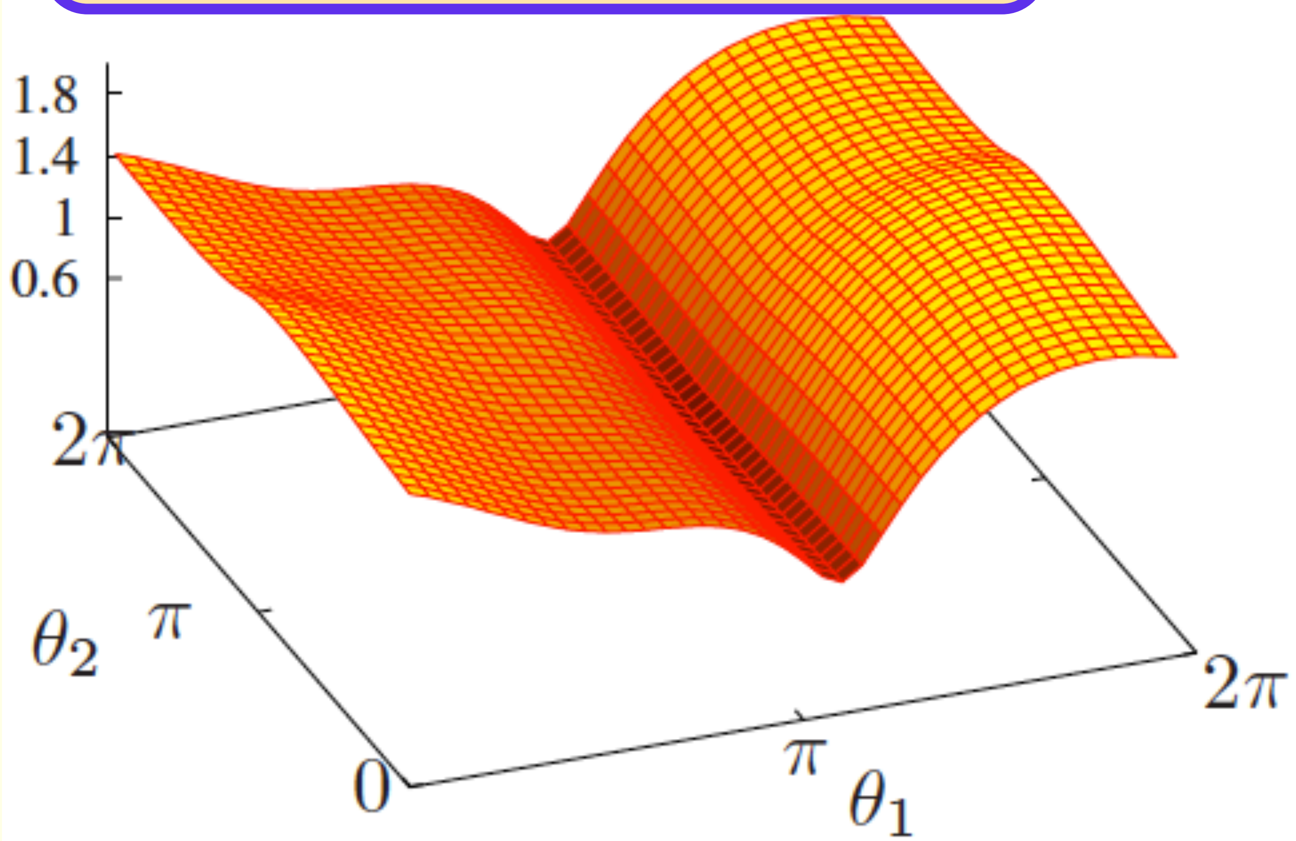
Protophases are distorted phases, this imitates nonlinearity in measurements: $\theta_{1,2} = \phi_{1,2} + \frac{1}{2}\sin(\phi_{1,2}) + \frac{1}{10}\cos(2\phi_{1,2})$





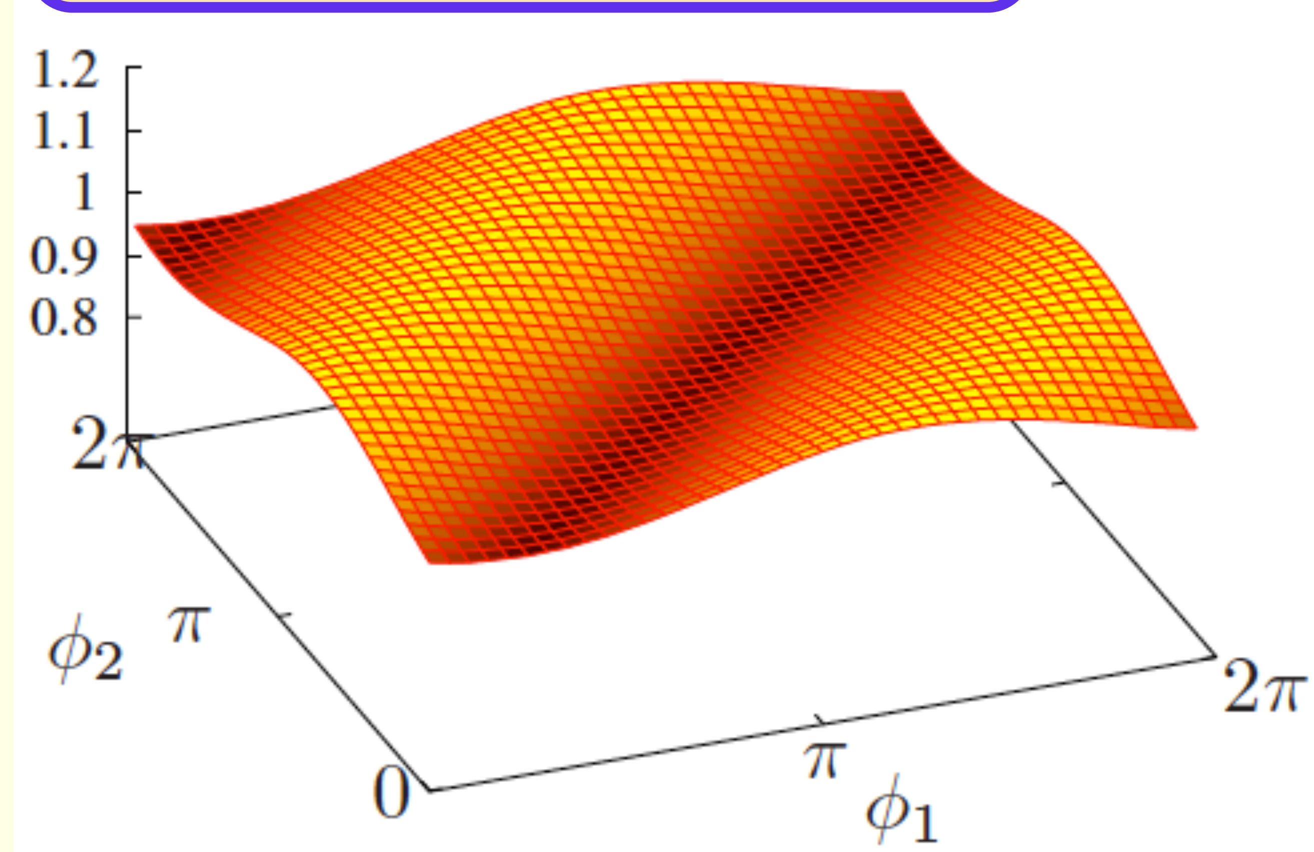
Why is the transformation $\theta \to \phi$ important?

Function of protophases



Is dominated by the spurious dependence on the own phase!

Function of phases



Is correct!