DAMOCO Toolbox
Brief illustration to the theory

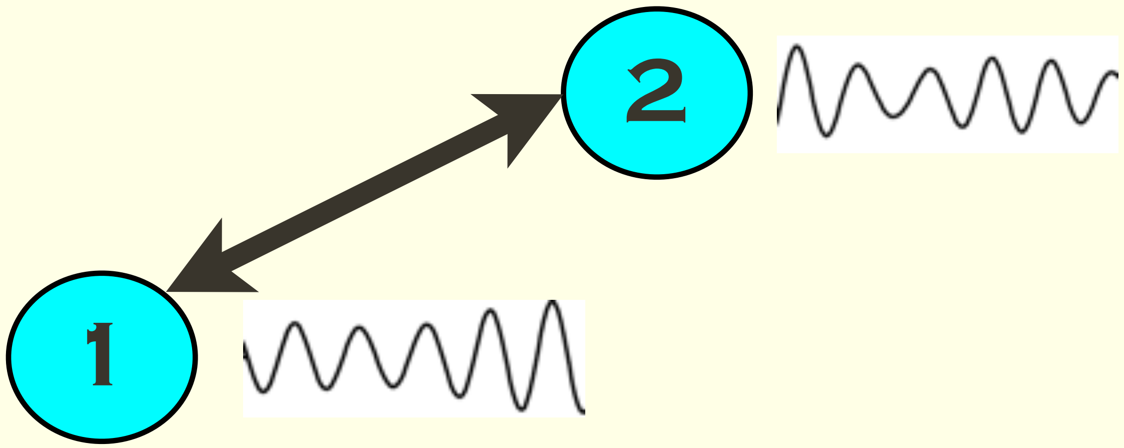
Direction of coupling and its quantification

Michael Rosenblum

Bjoern Kralemann **Arkady Pikovsky**

Institute of Physics and Astronomy
Potsdam University, Germany

Two oscillators



Suppose we have reconstructed the model of phase dynamics

$$\dot{\varphi}_1 = \omega_1 + q_1(\varphi_1, \varphi_2)$$
$$\dot{\varphi}_2 = \omega_2 + q_2(\varphi_1, \varphi_2)$$

Norm $||q_1||$ quantifies the strength of action $2 \to 1$

Relative measure $c_1 = \dfrac{||q_1||}{\omega_1}$, similarly $c_2 = \dfrac{||q_2||}{\omega_2}$

Directionality index $d = \dfrac{c_2 - c_1}{c_2 + c_1}, -1 \leq d \leq 1$

$d = -1$: second unit drives the first one

$d = 1$: first unit drives the second one

$-1 < d < 1$: bidirectional driving

Network of oscillators

Coupling functions in terms of Fourier coefficients:

$$\frac{d\varphi_k}{dt} = \omega_k + q_k(\varphi_1, \varphi_2, \ldots, \varphi_N)$$

$$= \sum_{l_1,\ldots,l_N} \mathcal{F}^{(k)}_{l_1,\ldots,l_N} \exp\left(il_1\varphi_1 + il_2\varphi_2 + \ldots + l_N\varphi_N\right)$$

Norm of the coupling function q_k quantifies effect of the rest of the network on oscillator k

Action of particular oscillator $j \to k$

Partial norm $\mathcal{N}^2_{k\leftarrow j} = \sum_{l_k, l_j \neq 0} \left| \mathcal{F}^{(k)}_{0,\ldots,l_k,0,\ldots,l_j,0,\ldots} \right|^2$

Notice: generally $\mathcal{N}_{k\leftarrow j} \neq \mathcal{N}_{k\rightarrow j}$, hence it quantifies directional connectivity