

# Thermodynamics of heating a room

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A room is not heated by increasing its internal energy but by decreasing its entropy due to the fact that during heating, the volume and pressure remain constant and air is expelled. We first present a simple solution treating the air in the room as an ideal gas. We calculate the differential entropy change and heat transfer and give numbers for a typical room including estimates of heat loss through windows and walls. We also demonstrate the power of thermodynamics to derive the entropy and internal energy changes for any gas. © 2011 American Association of Physics Teachers. [DOI: 10.1119/1.3488987]

## I. INTRODUCTION

In 1938, Emden<sup>1</sup> published a short article with the title, “Why do we have winter heating?”<sup>2</sup> He starts by giving two answers:

“The layman will answer: ‘To make the room warmer.’ The student of thermodynamics will perhaps express it thus: ‘To import the lacking (internal thermal) energy.’ If so, then the layman’s answer is right, the scientist’s is wrong.”

Emden then proved his assertion with a simple argument based solely on regarding air as an ideal gas. Sommerfeld included this thermodynamic problem in his textbook on thermodynamics.<sup>3</sup> Many years later, it was briefly discussed in two short communications.<sup>4,5</sup> The lack of exposure and interest in this problem is surprising, considering the importance of heating of dwellings in everyday life and how instructive this example is. Over the years, we have jokingly asked many physicists Emden’s question, with the result that embarrassingly few came up with the correct answer. We have therefore decided to revisit this problem more formally and also consider situations where the ideal gas law does not hold. It is also fitting to pay tribute to Emden,<sup>1</sup> an eminent scientist and thermodynamicist who first raised the issue and who pioneered the application of thermodynamics in atmospheric science and astrophysics.

What happens when you “heat” a room to increase its temperature? The quick answer is you turn on a heater, thus increasing the internal energy and hence raising the temperature. Although this answer may apply to some rooms, it is not so for rooms in which we live. We analyze this answer by first paying careful attention to what is the system and what are its enclosing walls before we proceed with a more realistic answer.

Energy transfer by heating a system will be used solely to increase its internal energy only if its enclosing walls are rigid (otherwise some of the energy transferred by heating could be used to do work on the outside) and the walls are impermeable (otherwise air will escape taking some of the thermal energy with it). Rooms with rigid and impermeable walls exist, but hardly qualify as living rooms for obvious reasons.

The thermodynamic system of interest consists of the air in the room. The walls surrounding a room can be well characterized as rigid because the thermal expansion of the walls

is negligible so that the volume of the room is constant. We will also assume that the walls are adiabatic. We will deal with energy loss by conduction through walls and windows later. The walls of a room are approximately impermeable except for the doors and windows. Thus, overall the walls must be treated as permeable so that air can escape and enter. Permeability implies that the air pressure inside and outside the room is the same and remains constant because anything we do to the room will hardly affect the world outside. Thus, our preliminary conclusion is that during the process of raising the temperature, the pressure and volume of the room remain constant, and the amount of air in the room (mole number) does not. These are the conditions for which we must formulate the solution.

## II. SIMPLE SOLUTION: IDEAL GAS

For the conditions applicable to human habitation, we can treat the air as an ideal gas of  $n$  moles contained in a volume  $V$  at pressure  $P$  so that  $PV=nRT$ . With  $P$  and  $V$  both constant, the internal energy  $U=nc_vT=c_vPV/R$  remains constant, where  $c_v$  is the molar specific heat at constant volume. Thus, raising the temperature of the room can only be done by expelling air:  $T$  goes up and  $n=U/(c_vT)$  goes down to keep  $U$  constant. Of the relevant extensive variables, the internal energy  $U$ , volume  $V$ , mole number  $n$ , and entropy  $S=S(U, V, n)$ , the only remaining thermodynamic property that can also change is the entropy. Because the entropy for an ideal gas is known (see, for example, Refs. 6 and 7), we could calculate its change directly at fixed  $U$  and fixed  $P$ . However, it is a better thermodynamic practice to start from differential changes. To simplify the calculation of the change of entropy, we fix  $V$  and  $U$  rather than  $P=UR/(c_vV)$ . Thus, we need to know

$$dS = \left( \frac{\partial S}{\partial T} \right)_{U,V} dT = \left( \frac{\partial S}{\partial n} \right)_{U,V} \left( \frac{\partial n}{\partial T} \right)_{U,V} dT \quad (1a)$$

$$= \left( -\frac{\mu}{T} \right) \left( -\frac{n}{T} \right) dT = \frac{\mu}{RT} \frac{PV}{T^2} dT. \quad (1b)$$

Equation (1b) follows from the mechanical equation of state of the ideal gas and the fundamental relation in the entropy representation

$$dS = (1/T)dU + (P/T)dV + (-\mu/T)dn \quad (2)$$

so that

$$\left(\frac{\partial S}{\partial n}\right)_{U,V} = -\frac{\mu}{T}. \quad (3)$$

The chemical potential of an ideal classical gas of molecules at room temperature is negative and is given by

$$\exp(\mu/RT) = N_A \frac{P}{RT} \lambda_{\text{th}}^3 Z_{\text{int}}^{-1}, \quad (4)$$

where  $N_A$  is Avogadro's number,  $\lambda_{\text{th}} = h/(2\pi mk_B T)^{1/2}$  is the thermal wavelength of the gas particles with average mass  $m$ , and  $Z_{\text{int}}$  is the molecular partition function for the vibrational and rotational degrees of freedom of the air molecules (see, for example, Ref. 7 or Ref. 8). The result is that as the temperature is raised in the room, the entropy decreases. Thus, the heat transfer,  $dQ = TdS$ , is also negative, that is, we do transfer energy to the outside to “heat” a room. The decrease in entropy, with an increase of temperature, can only occur if the quantity of air in the fixed volume decreases, that is, air must be expelled to the outside.

We can integrate Eq. (1b) to obtain the entropy change and also the energy transfer needed to raise the temperature of the air from  $T_i$  to  $T_f$ ,

$$\begin{aligned} \Delta S &= S_f - S_i \\ &= \int_{T_i}^{T_f} dS = -PV \left[ \frac{1}{T_f} \left( \frac{\mu_f}{RT_f} - \frac{5}{2} \right) - \frac{1}{T_i} \left( \frac{\mu_i}{RT_i} - \frac{5}{2} \right) \right], \end{aligned} \quad (5)$$

$$\begin{aligned} Q &= \int_{T_i}^{T_f} TdS \\ &= PV \left\{ \ln \left[ N_A \frac{P}{R} (h^2/2\pi mk_B)^{3/2} \right] - \frac{1}{2} c_P \ln(T_i T_f) \right\} \ln \frac{T_f}{T_i}. \end{aligned} \quad (6)$$

Here,  $c_P = c_V + R$  is the specific heat at constant pressure ( $c_P = 3.48R$  for dry air, that is, there is a contribution of  $\frac{1}{2}R$  to  $c_V$  for each of three translational degrees of freedom and two rotations, but vibrations are not excited at room temperature).

To obtain a sense for the magnitude of these changes, consider a  $4 \text{ m} \times 5 \text{ m} \times 2.5 \text{ m}$  room with volume  $V = 50 \text{ m}^3$  and atmospheric pressure whose temperature is raised from  $T_i = 273 \text{ K}$  to  $T_f = 300 \text{ K}$ . We have to expel thermal energy  $-Q \approx 10 \text{ MJ}$  or  $2.8 \text{ kW h}$ , provided we make the room completely air tight and thermally insulated as soon as the final temperature is reached. From the equation of state we have that  $n_f T_f = n_i T_i$  so that in this heating process about 10% of the air has been expelled. The entropy change is  $\Delta S \approx -26 \text{ kJ/K}$ . To put this number into perspective, we note that the magnitude of this entropy change is about the same as that occurs by heating and transforming four liters of water completely into vapor at 1 atm pressure, for which about 9 MJ is required ( $\Delta S$  and  $Q$  positive).

As Sommerfeld and others have discussed,<sup>4,5</sup> although the internal energy remains constant, the molar internal energy  $u = U/n$  increases by

$$|u_f - u_i| = c_V (T_f - T_i). \quad (7)$$

Because the internal energy in an ideal gas is purely kinetic, this increase implies that at higher temperature the kinetic energy of the gas molecules is higher.

In addition to raising the temperature in a room, we also have to raise the temperature in the walls. For a thickness of about 1 cm, the total volume of gypsum plasterboard (common in a standard North American home) covering the walls (density of about  $600 \text{ kg/m}^3$ ) is about  $1 \text{ m}^3$ , which contains about  $n = 3400 \text{ mol}$  of calcium sulfate dihydrate. The specific heat of plasterboard is about  $1090 \text{ J kg}^{-1} \text{ K}^{-1}$ . Thus, the energy transfer needed to raise the temperature of the plasterboard by the same amount (27 K) is  $Q_{\text{PB}} = nc_V (T_f - T_i) \approx 0.31 \text{ kW h}$ , or about 1/10 of the heat transfer needed to raise the temperature of the air itself. In addition, we should keep in mind that there is energy loss by conduction and by convection through cracks and doors. To be specific, the rate of heat loss as measured by the  $U$ -factor (the inverse of the  $R$ -value) is  $0.5 \text{ W K}^{-1} \text{ m}^{-2}$  for a stud wall with R-20 fiberglass insulation,  $6.25 \text{ W K}^{-1} \text{ m}^{-2}$  for a single-glazed window, and  $2.8 \text{ W K}^{-1} \text{ m}^{-2}$  for a double-glazed window. Thus, for a  $2.5 \text{ m} \times 5 \text{ m}$  outside wall with a  $2 \text{ m}^2$  double-glazed window and a temperature difference of 20 K between the inside of the house and the outside we must supply about 240 W, or 6 kW h/day to compensate for the heat loss through conduction, provided there are no thermal leaks by convection.

### III. GENERAL SOLUTION

What happens at higher pressure and higher densities when the ideal gas approximation is not applicable? Likewise, what happens when we want to superheat steam? In such situations, the simplification that constant pressure and volume implies constant internal energy and volume cannot be used. A student of thermodynamics must learn to treat such problems in a systematic way without resorting to simplifying “tricks” as used in many books. Here, we follow the procedure outlined in Ref. 6, Chap. 7.3. The procedure is a good exercise in thermodynamic thinking and rigor.

We need the entropy change for a given temperature change at constant pressure and volume,

$$dS = \left(\frac{\partial S}{\partial T}\right)_{P,V} dT = \left(\frac{\partial S}{\partial n}\right)_{P,V} \left(\frac{\partial n}{\partial T}\right)_{P,V} dT. \quad (8)$$

We write

$$\left(\frac{\partial n}{\partial T}\right)_{P,V} = -\frac{(\partial V/\partial T)_{P,n}}{(\partial V/\partial n)_{T,P}}. \quad (9)$$

The numerator is the thermal expansion coefficient  $\alpha$  at constant pressure up to a factor of  $V$ ,

$$V\alpha = \left(\frac{\partial V}{\partial T}\right)_{P,n}. \quad (10)$$

For the denominator, we have the Maxwell relation

$$\left(\frac{\partial V}{\partial n}\right)_{T,P} = \frac{\partial^2 G}{\partial P \partial n} = \left(\frac{\partial \mu}{\partial P}\right)_{T,n} = \frac{V}{n}. \quad (11)$$

The last equality results from the fact that, for constant  $T$ , the Gibbs–Duhem equation

$$d\mu = -\frac{S}{n}dT + \frac{V}{n}dP \quad (12)$$

reduces to  $d\mu = (V/n)dP$ . We thus obtain

$$\left. \frac{\partial n}{\partial T} \right|_{P,V} = -n\alpha. \quad (13)$$

We turn next to the first factor in Eq. (8) and write

$$\left( \frac{\partial S}{\partial n} \right)_{P,V} = -\frac{(\partial P/\partial n)_{S,V}}{(\partial P/\partial S)_{V,n}} \quad (14)$$

to obtain the quantities that are specified to be the natural variables of the internal energy,  $U=U(S, V, n)$ . We thus obtain a useful Maxwell relation for the numerator

$$-\left( \frac{\partial P}{\partial n} \right)_{S,V} = \frac{\partial^2 U}{\partial V \partial n} = \left( \frac{\partial \mu}{\partial V} \right)_{S,n} \quad (15)$$

$$= -\frac{S}{n} \left( \frac{\partial T}{\partial V} \right)_{S,n} + \frac{V}{n} \left( \frac{\partial P}{\partial V} \right)_{S,n}, \quad (16)$$

where we again used Eq. (12). For the denominator in Eq. (14), we have another Maxwell relation

$$\left( \frac{\partial P}{\partial S} \right)_{V,n} = -\frac{\partial^2 U}{\partial V \partial S} = -\left( \frac{\partial T}{\partial V} \right)_{S,n}, \quad (17)$$

so that

$$\left( \frac{\partial S}{\partial n} \right)_{P,V} = \frac{S}{n} - \frac{V}{n} \left( \frac{\partial P}{\partial V} \right)_{S,n} \left( \frac{\partial V}{\partial T} \right)_{S,n} = \frac{S}{n} - \frac{V}{n} \left( \frac{\partial P}{\partial T} \right)_{S,n} \quad (18a)$$

$$= \frac{S}{n} - \frac{V}{n} \left( \frac{\partial S}{\partial T} \right)_{P,n} \left( \frac{\partial T}{\partial V} \right)_{P,n} = \frac{S}{n} - \frac{1}{T} \frac{c_P}{\alpha}. \quad (18b)$$

Hence, we obtain for Eq. (8),

$$\left( \frac{\partial S}{\partial T} \right)_{P,V} = -n\alpha \left( s - \frac{c_P}{\alpha T} \right). \quad (19)$$

$$dS = -n\alpha \left( s - \frac{c_P}{\alpha T} \right) dT, \quad (20)$$

where  $s=S/n$  is the molar entropy. If we express the Euler relation for molar quantities,  $s=u/T+Pv/T-\mu/T$ , we can easily show that this general result agrees with Eq. (1b) for an ideal gas. The quantities  $s$ ,  $\alpha$ , and  $c_P$  are well defined functions of temperature and pressure and are tabulated for many gases of interest.

What happens to the internal energy under conditions when  $P$  and  $V$  are held constant? We obtain

$$dU = \left( \frac{\partial U}{\partial T} \right)_{P,V} dT = -n\alpha \left( h - \frac{c_P}{\alpha} \right) dT, \quad (21)$$

where  $h=u+Pv$  is the molar enthalpy. For an ideal gas, Eq. (21) reduces to  $dU=0$  as required. If we use the definitions of  $\alpha$  and  $c_P$ , we can rewrite Eq. (20) as

$$dS = ns \left[ \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{P,n} - \frac{1}{S} \left( \frac{\partial S}{\partial T} \right)_{P,n} \right] dT, \quad (22)$$

which is the difference between the relative volume and entropy changes with temperature. Likewise, we obtain for Eq. (21),

$$dU = -nh \left[ \left( \frac{1}{V} \frac{\partial V}{\partial T} \right)_{P,n} - \frac{1}{H} \left( \frac{\partial H}{\partial T} \right)_{P,n} \right] dT. \quad (23)$$

To see how the nonideality of the gas affects the heating process, we need to integrate Eqs. (20) and (23) for a particular gas, which can be done numerically, for example, for water vapor using steam tables. However, a qualitative argument can be given that suggests no great surprises. We note that switching on interactions between the gas particles will decrease the expansion coefficient relative to an ideal gas because attractive forces will hinder expansion. Likewise, the entropy will decrease because attractive forces will result in short-ranged correlations. Hence, we expect that the term in brackets in Eq. (22) will remain negative. For water vapor at atmospheric pressure and 100 °C, the change  $(\partial s/\partial T)_P$  is  $-0.034R$  compared with the value for an ideal gas of  $-0.059R$ . The effect of saturated water vapor ( $P=0.01$  atm) at room temperature upon heating a room reduces the entropy loss marginally because the fraction of water vapor is only a few percent. A similar argument for the change in internal energy in Eq. (21) shows that it is marginally negative because the expelled air takes some of the internal energy with it, due to the attractive interactions between the molecules.<sup>10</sup>

#### IV. DISCUSSION

We have argued that to raise the temperature of a room with permeable walls, the relevant thermodynamic process has both volume and pressure constant. If we treat the air in the room as ideal (which is a very good approximation), its internal energy  $U=nc_V T=(c_V/R)PV$  remains constant, implying that raising the temperature is accompanied by expelling air to the outside and lowering the entropy. Thus, the thermal energy that is transferred to the room is also transferred to the outside where it is wasted. Converse statements apply if the room is cooled.

A skeptical reader might not be convinced and still believe that there may be an alternate scenario. Let us analyze two such possibilities. Because our analysis is based on the fact that the walls of the room are permeable (the windows and doors) so that air can escape, we now remove this assumption and investigate a hermetically sealed room.

For a sealed room,  $n$  is constant, and we have two options, either the volume of the room stays fixed as the temperature is raised or the pressure remains constant. For constant volume, the pressure will rise according to

$$dP = \left( \frac{\partial P}{\partial T} \right)_{V,n} dT = -\frac{(\partial V/\partial T)_{P,n}}{(\partial V/\partial P)_{T,n}} dT = \frac{\alpha}{\kappa_T} dT. \quad (24)$$

We treat the air as an ideal gas. Increasing its temperature at constant volume from  $T_i=273$  K to  $T_f=300$  K will raise the pressure from  $P_i=1$  atm=101.3 kPa to  $P_f=(T_f/T_i)P_i=1.1$  atm.<sup>11</sup> This increase might cause some health problems not to mention that large windows might explode. No work is done and the thermal energy transferred to the room is  $Q_{i \rightarrow f}=U_f-U_i=(c_V/R)V(P_f-P_i) \approx 0.35$  kW h.

For constant pressure, the volume will increase to  $V_f = (T_f/T_i)V_i$ , again a 10% increase, causing an unacceptable bulge in the windows and walls of the room. In this case, work is also done to move the walls (if that is possible), namely,  $W_{i \rightarrow f} = -P(V_f - V_i) \approx -0.14 \text{ kW h}$  accompanied by a heat transfer  $Q_{i \rightarrow f} = \Delta U - W_{i \rightarrow f} = (c_p/R)P(V_f - V_i) \approx 0.48 \text{ kW h}$ .

It is the entropy that plays the dominant role in thermodynamics, which sets it apart from mechanics. To emphasize this point, we finish with another quote from the conclusions of Emden's article:<sup>2</sup>

“As a student, I read with advantage a small book by F. Wald entitled ‘The Mistress of the World and her Shadow.’ These meant energy and entropy. In the course of advancing knowledge the two seem to me to have exchanged places. In the huge manufactory of natural processes, the principle of entropy occupies the position of manager, for it dictates the manner and method of the whole business, whilst the principle of energy merely does the book-keeping, balancing credits and debits.”

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<sup>1</sup>The Swiss physicist Robert Emden (1862–1940) was, successively, a professor of physics, meteorology, and astrophysics at the Technical University of Munich. His work *Gaskugeln: Anwendungen der Mechanischen Wärmetheorie auf Kosmologische und Meteorologische Probleme (Gaseous Spheres, The Application of Thermodynamics to Cosmological and Meteorological Problems)* (Teubner, Leipzig, 1907) and “Thermodynamics of celestial bodies,” *Encyklopadie der Mathematischen Wissenschaften* (Teubner, Leipzig, 1926), Vol. 6, pp. 373–532, laid the founda-

tions of atmospheric science and astrophysics and became the cornerstones of most textbooks on these subjects. He also laid the foundations of the theory of balloon flight and while considering atmospheric radiation, anticipated in 1921 Bose–Einstein statistics for photons in his work on “light quanta.” See R. Emden, “Lichtquanten,” *Phys. Z.* **22**, 513–517 (1921) For more information on this remarkable scientist, see [www.encyclopedia.com/doc/1G2-2830901313.html](http://www.encyclopedia.com/doc/1G2-2830901313.html).

<sup>2</sup>R. Emden, “Why do we have winter heating?,” *Nature (London)* **141**, 908–909 (1938).

<sup>3</sup>A. Sommerfeld, *Thermodynamik und Statistik*, Vorlesungen über Theoretische Physik Band 5 (Diederich'sche Verlagsbuchhandlung, Mainz, 1952) [translated by J. Kestin, *Thermodynamics and Statistical Mechanics*, Lectures on Theoretical Physics Vol. V (Academic, New York, 1956)].

<sup>4</sup>Z. Bilkadi, “When you heat your house does the thermal energy content increase?,” *J. Chem. Educ.* **49**, 493–494 (1972).

<sup>5</sup>J. A. Campbell, “Footnote to the house heating exemplum,” *J. Chem. Educ.* **50**, 365–366 (1973).

<sup>6</sup>Herbert B. Callen, *Thermodynamics and an Introduction to Thermostatistics*, 2nd ed. (Wiley, New York, 1985). This classic book and the following reference exemplify the approach to thermodynamics based on differentials.

<sup>7</sup>H. J. Kreuzer and I. Tamblyn, *Thermodynamics* (World Scientific, Singapore, 2010).

<sup>8</sup>J. Kestin and J. R. Dorfman, *A Course in Statistical Thermodynamics* (Academic, New York, 1971).

<sup>9</sup>For  $R$ -values of insulation and building materials, see, for example, [en.wikipedia.org/wiki/R-value\\_insulation](http://en.wikipedia.org/wiki/R-value_insulation).

<sup>10</sup>We point out some flaws in Ref. 4 so that readers do not become unnecessarily confused. This article has the subtitle, “Illustrating the first law of thermodynamics.” This subtitle is misleading because the energy in the room is conserved only within the model of an ideal gas. Otherwise, energy is conserved in the supersystem of room plus environment. Furthermore, the process of heating the room is driven by entropy. Along with this article are some comments by Wilbur B. Bridgman, which are misleading. We point out two major concerns: His Eq. (9) is mathematically inconsistent because the left-hand side is a differential and the right-hand side is a ratio of two differentials. The proper expression is our Eq. (21). Also the discussion of the crucial role of the reference state (emphasized in the statement at the end of his second to last paragraph “different conclusions are reached depending on the choice of reference state”) is incorrect because thermodynamic predictions cannot depend on the choice of a reference state, but are given by the integrals of differentials.

<sup>11</sup>The highest pressure ever recorded on Earth was 1.07 atm in Agata, Siberia on December 31, 1968 with a temperature of  $-58 \text{ }^\circ\text{C}$ . See [en.wikipedia.org/wiki/Atmosphere\\_pressure](http://en.wikipedia.org/wiki/Atmosphere_pressure).

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